MODELING AND SIMULATION OF THE VECTORIAL CONTROL SYSTEM WHICH CONTAIN AN EXTENDED GOPINATH OBSERVER

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ABSTRACT: *This paper presents a new flux and rotor resistance observer called an Extended Gopinath observer (EGO). The design of the EGO observer is done based on an adaptive mechanism using the notion of Popov hyperstability. The analysis of the new speed control system is released by simulation.*

KEY WORDS: *Extended Gopinath Observer (EGO), induction motor, control system.*

1. INTRODUCTION

This paper presents a new flux and rotor resistance observer called an Extended Gopinath observer (EGO). The design of the EGO observer is done based on an adaptive mechanism using the notion of Popov hyperstability [11].

Thus, this type of observer is included in the estimation methods based on an adaptation mechanism, along with the Extended Luenberger Observer (ELO) proposed by Kubota [4] and the Model Adaptive System (MRAS) observer proposed by Schauder [3].

In the second part of the paper is presented the analysis of the new speed control system, released by simulation.

2. THE GOPINATH EXTENDED OBSERVER

The EGO observer in composed of a low grade Gopinath rotoric flux observer and an adaptation mechanism used for the rotor resistance estimation.

The equations that define the rotoric flux Gopinath observer are [9]

$$\begin{cases} \dfrac{d}{dt}\hat{\underline{i}}_s = a_{11}^* \cdot \underline{i}_s + a_{12}^* \cdot \hat{\underline{\psi}}_r + b_{11}^* \cdot \underline{u}_s \\ \dfrac{d}{dt}\hat{\underline{\psi}}_r = a_{21}^* \cdot \underline{i}_s + a_{22}^* \cdot \hat{\underline{\psi}}_r + \underline{g} \cdot \left[\dfrac{d}{dt}\underline{i}_s - \dfrac{d}{dt}\hat{\underline{i}}_s \right] \end{cases} \quad (1)$$

where

$$a_{11}^* = -\left(\frac{1}{T_s^* \cdot \sigma^*} + \frac{1-\sigma^*}{T_r^* \cdot \sigma^*} \right); \; a_{12}^* = a_{13}^* - j \cdot a_{14}^* \cdot z_p \cdot \omega_r ;$$

$$a_{13}^* = \frac{L_m^*}{L_s^* \cdot L_r^* \cdot T_r^* \cdot \sigma^*}; \; a_{14}^* = \frac{L_m^*}{L_s^* \cdot L_r^* \cdot \sigma^*}; \; a_{21}^* = a_{31}^*;$$

$$a_{31}^* = \frac{L_m^*}{T_r^*} \; a_{22}^* = a_{33}^* + j \cdot z_p \cdot \omega_r ; \; a_{33}^* = -\frac{1}{T_r^*}; \; b_{11}^* = \frac{1}{L_s^* \cdot \sigma^*};$$

$$T_s^* = \frac{L_s^*}{R_s^*}; \; T_r^* = \frac{L_r^*}{\hat{R}_r}; \; \sigma^* = 1 - \frac{\left(L_m^*\right)^2}{L_s^* \cdot L_r^*}.$$

In the above relations I marked with "*" the identified electrical sizes of the induction motor.

The block diagram of the EGO is presented in Fig 1.

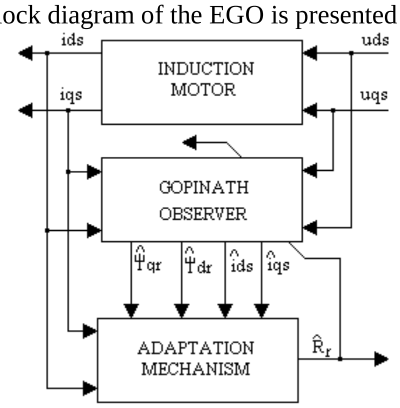

Fig.1 The Principle Schematic of the EGO Estimator

The essential element that determines the flux observer's stability, and also his lack of sensibility to the motor parameters variation, is a g gate, which is a complex number of the form: $\underline{g} = g_a + j \cdot g_b$.

In order to design this type of estimator we need to position the estimator's poles in the left Nyquist plane so that the estimator's stability is asured. The expresions g_a and g_b after the pole positioning are [9]

$$\begin{cases} g_a = \dfrac{1}{a_{14}^*} \cdot \left[\dfrac{\omega_r \cdot z_p \cdot \beta - \alpha \cdot a_{33}^*}{\left(\omega_r \cdot z_p\right)^2 + \left(a_{33}^*\right)^2} - 1 \right] \\ g_b = \dfrac{\omega_r \cdot z_p \cdot \alpha + a_{33}^* \cdot \beta}{\left(\omega_r \cdot z_p\right)^2 + \left(a_{33}^*\right)^2} \cdot \dfrac{1}{a_{14}^*} \end{cases} \quad (2)$$

The optimum position of the poles, the α and β values respectively, are obtained by the minimization of the induction motor's rotoric resistance variation over the stability of the flux observer. We get the following [9]:

$$\beta = 0; \alpha = k \cdot \sqrt{\left(\omega_r \cdot z_p\right)^2 + \left(a_{33}^*\right)^2}; k > 0 \quad (3)$$

In these conditions the Gopinath rotoric flux observer is completely determined.

Next, in order to determine the adaptation mechanism used to estimate the rotoric resistance, we will consider as a reference model the „statoric curents - rotoric fluxes" model of the induction motor and as an ajustable model, the model of the Gopinath rotoric flux observer. The equations mentioned above written under the input-state-output canonic form are:

Reference model:

$$\begin{cases} \dfrac{d}{dt} x = A \cdot x + B \cdot u \\ y = C \cdot \dfrac{d}{dt} x \end{cases} \quad (4)$$

Ajustable model:

$$\begin{cases} \dfrac{d}{dt} \hat{x} = \tilde{A} \cdot \hat{x} + \tilde{A}_1 \cdot x + B \cdot u + \tilde{G} \cdot \left(y - \hat{y}\right) \\ \hat{y} = C \cdot \dfrac{d}{dt} \hat{x} \end{cases} \quad (5)$$

where:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; \tilde{A} = \begin{bmatrix} 0 & a_{12}^* \\ 0 & a_{22}^* \end{bmatrix}; \tilde{A}_1 = \begin{bmatrix} a_{11}^* & 0 \\ a_{21}^* & 0 \end{bmatrix}; \tilde{G} = \begin{bmatrix} 0 \\ \underline{g} \end{bmatrix};$$

$$x = \begin{bmatrix} \underline{i}_s \\ \underline{\psi}_r \end{bmatrix}; \hat{x} = \begin{bmatrix} \hat{\underline{i}}_s \\ \hat{\underline{\psi}}_r \end{bmatrix}; u = \underline{u}_s; B = \begin{bmatrix} b_{11} \\ 0 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

In the above relations we marked with „~" the Gopinath estimator's matrices which are dependent upon the rotoric resistance, which in turn needs to be estimated based on the adaptation mechanism

Next, in order to determine the expresion that defines the adaptation mechanism we will asume that the identified electric sizes are identical with the real electric sizes of the induction motor. In other words:

$$a_{ij} = a_{ij}^*; i, j = 1,2 \text{ and } b_{11} = b_{11}^*$$

In order to build the adaptive mechanism, for start we will calculate the estimation error given by the difference:

$$e_X = x - \hat{x} \quad (6)$$

Derivation the relation (6) in relation with time and by using the relations (4) and (5) the relation (6) becomes:

$$\frac{d}{dt} e_X = \left(A - \tilde{A}_1\right) \cdot x - \tilde{A} \cdot \hat{x} - \tilde{G} \cdot C \cdot \frac{d}{dt} e_X \quad (7)$$

If the determinant $\det\left(I_2 + \tilde{G} \cdot C\right) \neq 0$ then it exists a unique inverse matrix $M = \left(I_2 + \tilde{G} \cdot C\right)^{-1}$ so that the expression (7) can be written like this:

$$\frac{d}{dt} e_X = M \cdot \left(A - \tilde{A}_1\right) \cdot e_X + M \cdot \left(A - \tilde{A}_1 - \tilde{A}\right) \cdot \hat{x} \quad (8)$$

Equation (8) describes a linear system defined by the term $M \cdot \left(A - \tilde{A}_1\right) \cdot e_X$ in inverse connection with a non linear system defined by the term $\Phi\left(e_y\right)$ which receives at input the error $e_y = C \cdot e_x$ between th e2 models and has at the output the term:

$$\rho = -M \cdot \left(A - \tilde{A}_1 - \tilde{A}\right) \cdot \hat{x} \quad (9)$$

The block diagram of the system that describes the dynamic evolution of the error between the state of the reference model and the state of the adjustable model is presented in Fig. 2.

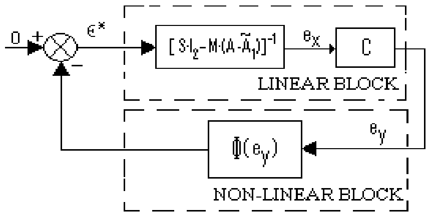

Fig. 2 The block diagram of the system (7)

As one may notice, this problem is frequently treated in the literature of the non-linear systems, being exactly the configuration of the Lure problem, and of one of the problems treated by Popov.

Considering, according to the Popov terminology, the non-linear block described by $\Phi\left(e_y\right)$ the integral input- output index associated to it is:

$$\eta\left(t_0, t_1\right) = \mathrm{Re}\left[\int_0^{t_1} e_y^T\left(t\right) \cdot \rho\left(t\right) dt\right] \quad (10)$$

In order for block to be hyper-stable a necessary condition is:

$$\eta\left(0, t_1\right) = \mathrm{Re}\left[\int_0^{t_1} e_y^T\left(t\right) \cdot \rho\left(t\right) dt\right] \geq -\gamma^2\left(0\right) \quad (11)$$

for any input-output combination and where $\gamma\left(0\right)$ is a positive constant.

In the above relation we marked with e_y^T the following expression:

$$e_y^T = \left[\bar{e}_y \quad 0\right] \quad (12)$$

obtained in order to keep the compatibility between the input and output dimensions, and $\bar{e}_y$ represents the conjugate of the complex variable e_y.

Under these circumstances, using the relation (9) the expression (11) becomes:

$$\eta\left(0, t_1\right) = -\mathrm{Re}\left[\int_0^{t_1} e_y^T\left(t\right) \cdot M \cdot \left(A - \tilde{A}_1 - \tilde{A}\right) \cdot \hat{x} dt\right] \geq -\gamma^2\left(0\right) \quad (13)$$

Next we asume that the error $M\cdot\left(A-\hat{A}_1-\hat{A}\right)$ is determined only by the rotor resistance estimation of the induction machine. In this case we may write:

$$M\cdot\left(A-\hat{A}_1-\hat{A}\right)=\left(R_r-\hat{R}_r\right)\cdot A_{er} \tag{14}$$

where: $A_{er}=\dfrac{1}{L_r}\cdot\begin{bmatrix} -\dfrac{1-\sigma}{\sigma} & a_{14} \\ L_m+\dfrac{1-\sigma}{\sigma}\cdot\underline{g} & -\left(1+\underline{g}\cdot a_{14}\right)\end{bmatrix}$.

For any positive derivable f function we can demonstrate the following inequality:

$$K_1\cdot\int_0^{t_1}\left(\frac{df}{dt}\cdot f\right)dt\geq-\frac{K_1}{2}\cdot f^2(0) \tag{15}$$

On the other hand, using the relation (14), the expression (13) becomes:

$$\eta(0,t_1)=-\mathrm{Re}\left\{\int_0^{t_1}\left[e_y^T(t)\cdot A_{er}\cdot\hat{x}\cdot\left(R_r-\hat{R}_r\right)\right]dt\right\}\geq-\gamma^2(0) \tag{16}$$

By combining the relations (15) and (16) we can write the following relations:

$$\begin{cases} f=R_r-\hat{R}_r \\ -\mathrm{Re}\left(e_y^T\cdot A_{er}\cdot\hat{x}\right)=K_1\cdot\dfrac{df}{dt}\end{cases} \tag{17}$$

Because K_1 is a constant and then, in case of a slower R_r parameter variation related to the adaptive law, we can write:

$$\hat{R}_r=k_i\cdot\int\mathrm{Re}\left(e_y^T\cdot A_{er}\cdot\hat{x}\right)dt \tag{18}$$

After replacing the variables that define the above expression (18) and taking into account the arbitrary nature of the K_i positive constant we obtain:

$$\hat{R}_r=k_i\cdot\int\left[e_{yd}\cdot\left(\hat{\psi}_{dr}-L_m\cdot\hat{i}_{ds}\right)+e_{yq}\cdot\left(\hat{\psi}_{qr}-L_m\cdot\hat{i}_{qs}\right)\right]dt \tag{19}$$

where $e_{yd}=i_{ds}-\hat{i}_{ds}$ and $e_{yq}=i_{qs}-\hat{i}_{qs}$.

Sometimes, insted of the adaptation law (19) we can use the following form:

$$\begin{aligned}\hat{R}_r=K_R\cdot&\left[e_{yd}\cdot\left(\hat{\psi}_{dr}-L_m\cdot\hat{i}_{ds}\right)+e_{yq}\cdot\left(\hat{\psi}_{qr}-L_m\cdot\hat{i}_{qs}\right)\right]+\\ &+k_i\cdot\int\left[e_{yd}\cdot\left(\hat{\psi}_{dr}-L_m\cdot\hat{i}_{ds}\right)+e_{yq}\cdot\left(\hat{\psi}_{qr}-L_m\cdot\hat{i}_{qs}\right)\right]dt\end{aligned} \tag{20}$$

From the above relation we ca observe that a new proportional component apears from the desire to have 2 coefficients that can control the speed estimation dynamics. This fact isn't always necesary because we can obtain very good results by using only expresion (19). Thus expresion (20) represents the general formula of the adaptation mechanism where K_R represents the proportionality constant and $K_i=K_R/T_R$; where T_R represents the integration time of the proportional-integral controller that defines the adaptation mechanism.

3. THE MATHEMATICAL DESCRIPTION OF THE VECTOR CONTROL SYSTEM

The block diagram of the control system of the mechanical angular speed ω_r of the induction motor with a discreet orientaion after the rotoric flux (DFOC) is presented in Fig. 3.

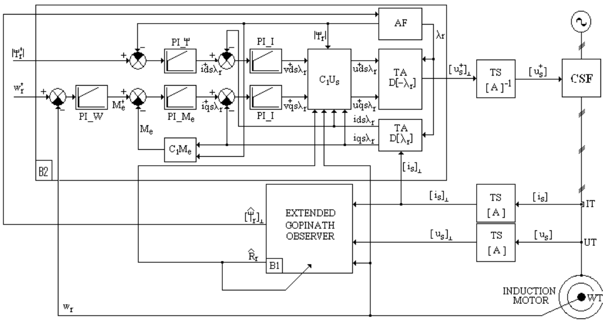

Fig. 3 The block diagram of the vector control system which contains an EGO loop.

In Fig. 3 we marked with B2 the control block of the speed control system with direct orientation after the rotoric flux (DFCO) and with B1 the extended Gopinath estimator block (EGO).

In order to mathematically describe the DFOC control system the following hypotheses have been considered:

- The static frequency converter (CSF) is assumed to contain a tension inverter.
- The static frequency converter is considered ideal so that the vector of the command measures is considered to be the entry vector of the induction motor.
- The dynamic measure transducers are considered ideal.

The mathematical model of the vector control system will be written in an $d\lambda_e-q\lambda_e$ axis reference bounded to the stator current.

Some of the equations that define the vector control system are given by the elements which compose the field orientation block and consist of:

- stator voltage decoupling block (C_1U_s):

$$\begin{cases} u^*_{ds\lambda_r}=\dfrac{1}{b^*_{11}}\cdot\left[b^*_{11}\cdot v^*_{ds\lambda_r}-a^*_{13}\cdot|\psi_r|-a^*_{31}\cdot\dfrac{i^2_{qs\lambda_r}}{|\psi_r|}-z_p\cdot\omega_r\cdot i_{qs\lambda_r}\right] \\ u^*_{qs\lambda_r}=\dfrac{1}{b^*_{11}}\cdot\left[b^*_{11}\cdot v^*_{qs\lambda_r}+a^*_{14}\cdot z_p\cdot\omega_r\cdot|\psi_r|+a^*_{31}\cdot\dfrac{i_{ds\lambda_r}\cdot i_{qs\lambda_r}}{|\psi_r|}+z_p\cdot\omega_r\cdot i_{ds\lambda_r}\right]\end{cases} \tag{21}$$

- PI flux controller (PI_ψ) defined by the K_ψ proportionality constant and the T_ψ integration time:

$$\begin{cases} \dfrac{dx_6}{dt}=\psi_r^*-|\psi_r| \\ i^*_{ds\lambda_r}=\dfrac{K_\psi}{T_\psi}\cdot x_6+K_\psi\cdot\left(\psi_r^*-|\psi_r|\right)\end{cases} \tag{22}$$

- torque PI controller (PI_Me) defined by the K_M proportionality constant and the T_M integration time:

$$\begin{cases} \dfrac{dx_7}{dt}=M_e^*-M_e \\ i^*_{qs\lambda_r}=\dfrac{K_M}{T_M}\cdot x_7+K_M\cdot\left(M_e^*-M_e\right)\end{cases} \tag{23}$$

- mechanical angular speed PI controller (PI_W) defined by the K_ω proportionality constant and the T_ω integration time:

$$\begin{cases} \dfrac{dx_8}{dt} = \omega_r^* - \omega_r \\ M_e^* = \dfrac{K_\omega}{T_\omega} \cdot x_8 + K_\omega \cdot \left(\omega_r^* - \omega_r\right) \end{cases} \tag{24}$$

- current PI controller (PI_I) defined by the K_i proportionality constant and the T_i integration time:

$$\begin{cases} \dfrac{dx_9}{dt} = i^*_{ds\lambda_r} - i_{ds\lambda_r} \\ v^*_{ds\lambda_r} = \dfrac{K_i}{T_i} \cdot x_9 + K_i \cdot \left(i^*_{ds\lambda_r} - i_{ds\lambda_r}\right) \end{cases} \tag{25}$$

$$\begin{cases} \dfrac{dx_{10}}{dt} = i^*_{qs\lambda_r} - i_{qs\lambda_r} \\ v^*_{qs\lambda_r} = \dfrac{K_i}{T_i} \cdot x_{10} + K_i \cdot \left(i^*_{qs\lambda_r} - i_{qs\lambda_r}\right) \end{cases} \tag{26}$$

- Flux analyzer (AF):

$$\begin{cases} \left|\psi_r\right| = \sqrt{\hat{\psi}^2_{dr} + \hat{\psi}^2_{qr}} \\ \sin\lambda_r = \dfrac{\hat{\psi}_{qr}}{\left|\psi_r\right|} ; \cos\lambda_r = \dfrac{\hat{\psi}_{dr}}{\left|\psi_r\right|} \end{cases} \tag{27}$$

- The calculate of the torque block (C_1M_e):

$$M_e = K_a \cdot \left|\psi_r\right| \cdot i_{qs\lambda_r} \tag{28}$$

The other equations that define the mathematic model of the speed's vectorial control system are:

- The equations that define the stator currents – rotor fluxes mathematical model of the induction motor; 4 equations defined based on the first relation in the canonic system (4) to which we can add the induction machine's motion equation defined by the following expression:

$$\frac{d}{dt}\omega_r = K_{m_1} \cdot \left[\psi_{dr} \cdot i_{qs} - \psi_{qr} \cdot i_{ds}\right] - K_{m_2} \cdot \omega_r - K_{m_3} \cdot M_r \tag{29}$$

where $K_{m_1} = \dfrac{3}{2} \cdot \dfrac{z_p}{J} \cdot \dfrac{L_m}{L_r}$; $K_{m_2} = \dfrac{F}{J}$; $K_{m_3} = \dfrac{1}{J}$.

The equations that define the extended Gopinath observer defined by the 4 relations that can be written based on system (1) with the equation that defines the speed adaptation mechanism (20). Expresion (20) can also be written like below:

$$\begin{cases} \dfrac{d}{dt}x_{15} = e_{yd} \cdot \left(\hat{\psi}_{dr} - L_m \cdot \hat{i}_{ds}\right) + e_{yq} \cdot \left(\hat{\psi}_{qr} - L_m \cdot \hat{i}_{qs}\right) \\ \hat{R}_r = \dfrac{K_R}{T_R} \cdot x_{15} + K_R \cdot \left[e_{yd} \cdot \left(\hat{\psi}_{dr} - L_m \cdot \hat{i}_{ds}\right) + e_{yq} \cdot \left(\hat{\psi}_{qr} - L_m \cdot \hat{i}_{qs}\right)\right] \end{cases} \tag{30}$$

All these expressions form a 15 differential equations system with 15 unknown values. In order to offer a coerent presentation of this differential equations system, we have used the following notations:

The state vector of the control system will be

$$x = \left[x_i\right]_{i=\overline{1,15}} \tag{31}$$

where: $x_1 = i_{ds\lambda_e}$; $x_2 = i_{qs\lambda_e}$; $x_3 = \psi_{dr\lambda_e}$; $x_4 = \psi_{qr\lambda_e}$; $x_5 = \omega_r$; $x_{11} = \hat{i}_{ds\lambda_e}$; $x_{12} = \hat{i}_{qs\lambda_e}$; $x_{13} = \hat{\psi}_{dr\lambda_e}$; $x_{14} = \hat{\psi}_{qr\lambda_e}$.

The input vector of the of the control system will be

$$u = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T \tag{32}$$

where $u_1 = \psi_r^*$; $u_2 = \omega_r^*$; $u_3 = M_r$.

Under these circumstances the 15 differential equations system that define the mathematical model of the vector control system can be written as follows

$$\frac{d}{dt}x = f(x,u) \tag{33}$$

where $f(x,u) = \left[f_i(x,u)\right]_{i=\overline{1,15}}$ and the $f_i = f_i(x,u)$ functions are:

$$f_1(x,u) = a_{11} \cdot x_1 + \omega_e \cdot x_2 + a_{13} \cdot x_3 + a_{14} \cdot z_p \cdot x_5 \cdot x_4 + b_{11} \cdot u_a \tag{34}$$

$$f_2(x,u) = -\omega_e \cdot x_1 + a_{11} \cdot x_2 - a_{14} \cdot z_p \cdot x_5 \cdot x_3 + a_{13} \cdot x_4 + b_{11} \cdot u_b \tag{35}$$

$$f_3(x,u) = a_{31} \cdot x_1 + a_{33} \cdot x_3 + \left(\omega_e - z_p \cdot x_5\right) \cdot x_4 \tag{36}$$

$$f_4(x,u) = a_{31} \cdot x_2 - \left(\omega_e - z_p \cdot x_5\right) \cdot x_3 + a_{33} \cdot x_4 \tag{37}$$

$$f_5(x,u) = K_{m_1} \cdot \left[x_3 \cdot x_2 - x_4 \cdot x_1\right] - K_{m_2} \cdot x_5 - K_{m_3} \cdot u_3 \tag{38}$$

$$f_6(x,u) = u_1 - g_1 \tag{39}$$

$$f_7(x,u) = \frac{K_\omega}{T_\omega} \cdot x_8 + K_\omega \cdot \left(u_2 - g_2\right) - K_a \cdot g_3 \tag{40}$$

$$f_8(x,u) = u_2 - g_2 \tag{41}$$

$$f_9(x,u) = \frac{K_\psi}{T_\psi} \cdot x_6 + K_\psi \cdot \left(u_1 - g_1\right) - \frac{g_4}{g_1} \tag{42}$$

$$f_{10}(x,u) = \frac{K_M}{T_M} \cdot x_7 + K_M \cdot f_7(x,u) - \frac{g_3}{g_1} \tag{43}$$

$$f_{11}(x,u) = a^*_{11} \cdot x_1 + \omega_e \cdot x_2 + a^*_{13} \cdot x_{13} + a^*_{14} \cdot z_p \cdot g_2 \cdot x_{14} + b^*_{11} \cdot u_a \tag{44}$$

$$f_{12}(x,u) = -\omega_e \cdot x_1 + a^*_{11} \cdot x_2 - a^*_{14} \cdot z_p \cdot g_2 \cdot x_{13} + a^*_{13} \cdot x_{14} + b^*_{11} \cdot u_b \tag{45}$$

$$\begin{aligned} f_{13}(x,u) = {} & b_1 \cdot x_{13} + b_2 \cdot x_{14} - g_a \cdot b^*_{11} \cdot u_a + g_b \cdot b^*_{11} \cdot u_b + \\ & + b_3 \cdot x_1 + b_4 \cdot x_2 + g_a \cdot f_1(x,u) - g_b \cdot f_2(x,u) \end{aligned} \tag{46}$$

$$\begin{aligned} f_{14}(x,u) = {} & -b_2 \cdot x_{13} + b_1 \cdot x_{14} - g_b \cdot b^*_{11} \cdot u_a - g_a \cdot b^*_{11} \cdot u_b - \\ & - b_4 \cdot x_1 + b_3 \cdot x_2 + g_b \cdot f_1(x,u) + g_a \cdot f_2(x,u) \end{aligned} \tag{47}$$

$$f_{15}(x,u) = e_{yd} \cdot \left(x_{13} - L^*_m \cdot x_{11}\right) + e_{yq} \cdot \left(x_{14} - L^*_m \cdot x_{12}\right) \tag{48}$$

where

$g_1 = \sqrt{x^2_{13} + x^2_{14}}$; $g_2 = x_5$; $g_3 = x_2 \cdot x_{13} - x_1 \cdot x_{14}$;

$g_4 = x_1 \cdot x_{13} + x_2 \cdot x_{14}$; $b_1 = a^*_{33} - g_a \cdot a^*_{13} - g_b \cdot a^*_{14} \cdot z_p \cdot g_2$;

$g_5 = \dfrac{K_R}{T_R} \cdot x_{15} + K_R \cdot \left[e_{yd} \cdot \left(x_{13} - L^*_m \cdot x_{11}\right) + e_{yq} \cdot \left(x_{14} - L^*_m \cdot x_{12}\right)\right]$;

$b_2 = g_b \cdot a^*_{13} - \left(1 + g_a \cdot a^*_{14}\right) \cdot z_p \cdot g_2 + \omega_e$;

$b_3 = a^*_{31} - g_a \cdot a^*_{11} - g_b \cdot \omega_e$; $b_4 = g_b \cdot a^*_{11} - g_a \cdot \omega_e$;

$\omega_e = z_p \cdot g_2 + a^*_{31} \cdot \dfrac{x_2}{x_{13}}$; $\alpha = k \cdot \sqrt{\left(g_2 \cdot z_p\right)^2 + \left(a^*_{33}\right)^2}$;

$g_a = \dfrac{-1}{a^*_{14}} \cdot \left[1 + \dfrac{\alpha \cdot a^*_{33}}{\left(g_2 \cdot z_p\right)^2 + \left(a^*_{33}\right)^2}\right]$;

$$g_b = \frac{g_2 \cdot z_p \cdot \alpha}{\left(g_2 \cdot z_p\right)^2 + \left(a_{33}^*\right)^2} \cdot \frac{1}{a_{14}^*};$$

$$v_a = \frac{K_i}{T_i} \cdot x_9 + K_i \cdot f_9(x,u); \; v_b = \frac{K_i}{T_i} \cdot x_{10} + K_i \cdot f_{10}(x,u);$$

$$h_1 = a_{13}^* \cdot g_1 - a_{31}^* \cdot \frac{g_3^2}{g_1^3} - z_p \cdot g_2 \cdot \frac{g_3}{g_1};$$

$$h_2 = a_{14}^* \cdot z_p \cdot g_2 \cdot g_1 + a_{31}^* \cdot \frac{g_3 \cdot g_4}{g_1^3} + z_p \cdot g_2 \cdot \frac{g_4}{g_1};$$

$$u_a = \frac{1}{b_{11}^* \cdot g_1} \cdot \left[x_{13} \cdot \left(b_{11}^* \cdot v_a - h_1 \right) - x_{14} \cdot \left(b_{11}^* \cdot v_b + h_2 \right) \right];$$

$$u_b = \frac{1}{b_{11}^* \cdot g_1} \cdot \left[x_{14} \cdot \left(b_{11}^* \cdot v_a - h_1 \right) + x_{13} \cdot \left(b_{11}^* \cdot v_b + h_2 \right) \right].$$

$$e_{yd} = x_1 - x_{11} \; ; e_{yq} = x_2 - x_{12};$$

$$a_{11} = -\left(\frac{1}{T_s \cdot \sigma} + \frac{1-\sigma}{T_r \cdot \sigma} \right) \; ; \; a_{13} = \frac{L_m}{L_s \cdot L_r \cdot T_r \cdot \sigma} \; ;$$

$$a_{14} = \frac{L_m}{L_s \cdot L_r \cdot \sigma} \; ; \; a_{31} = \frac{L_m}{T_r} \; ; \; a_{33} = -\frac{1}{T_r} \; ; \; b_{11} = \frac{1}{L_s \cdot \sigma};$$

$$T_s = \frac{L_s}{R_s} \; ; \; T_r = \frac{L_r}{R_r} \; ; \; \sigma = 1 - \frac{L_m^2}{L_s \cdot L_r}; \; K_a = \frac{3}{2} \cdot z_p \cdot \frac{L_m^*}{L_r^*};$$

$$a_{11}^* = -\left(\frac{1}{T_s^* \cdot \sigma^*} + \frac{1-\sigma^*}{T_r^* \cdot \sigma^*} \right); a_{13}^* = \frac{L_m^*}{L_s^* \cdot L_r^* \cdot T_r^* \cdot \sigma^*};$$

$$a_{14}^* = \frac{L_m^*}{L_s^* \cdot L_r^* \cdot \sigma^*}; a_{31}^* = \frac{L_m^*}{T_r^*}; a_{33}^* = -\frac{1}{T_r^*}; b_{11}^* = \frac{1}{L_s^* \cdot \sigma^*};$$

$$T_s^* = \frac{L_s^*}{R_s^*}; T_r^* = \frac{L_r^*}{g_5}; \; \sigma^* = 1 - \frac{\left(L_m^*\right)^2}{L_s^* \cdot L_r^*}.$$

Under these circumstances the mathematical model of the speed vector control system is fully determined as being defined by the non-linear differential equations system given by (33) whose initial condition is $x(0) = 0$.

4. CONTROL SYSTEM ANALYSIS

In order to accomplish the above mentioned control system analysis, we shall consider an induction motor with a short-circuited rotor having the following electrical and mechanical parameters:

- electrical parameters

$R_s = 0.371\left[\Omega\right]$; $R_r = 0.415\left[\Omega\right]$; $L_s = 0.08694\left[\mathrm{H}\right]$; $L_r = 0.08762\left[\mathrm{H}\right]$; $L_m = 0.08422\left[\mathrm{H}\right]$.

- mechanical parameters

$z_p = 2$; $J = 0.15\left[\mathrm{kg \cdot m^2}\right]$; $F = 0.005\left[\frac{\mathrm{N \cdot m \cdot s}}{\mathrm{rad}}\right]$

On the other hand, following the controllers tuning within the speed control system the following constants have been obtained:

$$K_\psi = 501.3834; \; T_\psi = \frac{K_\psi}{2374.7};$$

$$K_i = 5.9881; \; T_i = \frac{K_i}{754.4176};$$

$$K_M = 10.1988; \; T_M = \frac{K_M}{1020};$$

$$K_\omega = 10; \; T_\omega = \frac{K_\omega}{350}; \; K_R = 6; \; T_R = \frac{K_R}{4000}.$$

In the relations above, K_R represents the proportionality constant and T_R represents the integration time of the PI control from the speed estimator designed based on the Popov hyperstability.

Next, the performances of the extended Gopinath estimator are presented in a variety of functional conditions.

Thus the image below will present the graphics for the real and estimated rotors fluxes and also the graphics for the imposed speed, real speed and the estimated speed for small, medium and large imposed speeds.

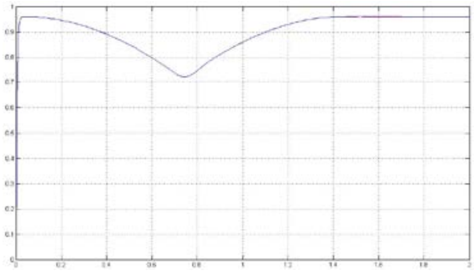

Fig. 4 ψ_{dr} real flux compared to the $\hat{\psi}_{dr}$ estimated flux: $\omega_r^* = 5 \cdot \frac{\pi}{30}\left[\frac{\mathrm{rad}}{\mathrm{s}}\right]; M_r = 0$.

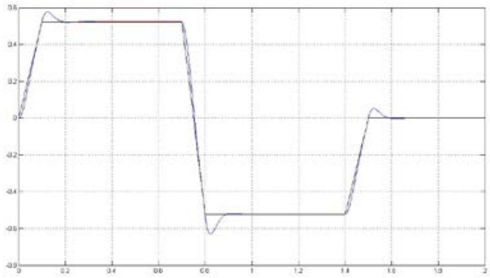

Fig. 5 ω_r real speed compared to the $\hat{\omega}_r$ estimated speed and reference speed $\omega_r^* = 5 \cdot \frac{\pi}{30}\left[\frac{\mathrm{rad}}{\mathrm{s}}\right]; M_r = 0$

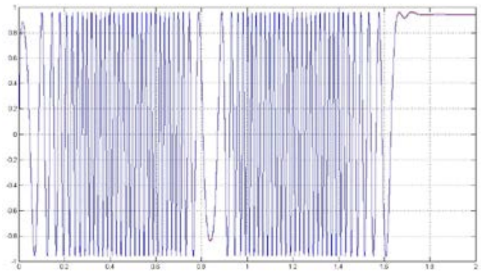

Fig. 6 ψ_{dr} real flux compared to the $\hat{\psi}_{dr}$ estimated flux: $\omega_r^* = 1500 \cdot \frac{\pi}{30}\left[\frac{\mathrm{rad}}{\mathrm{s}}\right]; M_r = 0$

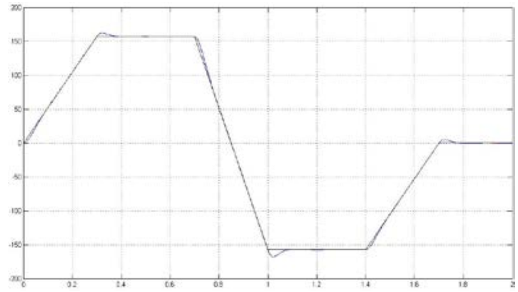

Fig. 6 ω_r real speed compared to the $\hat{\omega}_r$ estimated speed and reference speed: $\omega_r^* = 1500 \cdot \frac{\pi}{30}\left[\frac{\text{rad}}{\text{s}}\right]; M_r = 0$

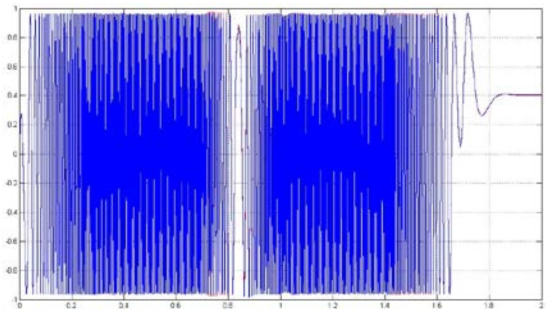

Fig. 7 ψ_{dr} real flux compared to the $\hat{\psi}_{dr}$ estimated flux: $\omega_r^* = 3000 \cdot \frac{\pi}{30}\left[\frac{\text{rad}}{\text{s}}\right]; M_r = 0$

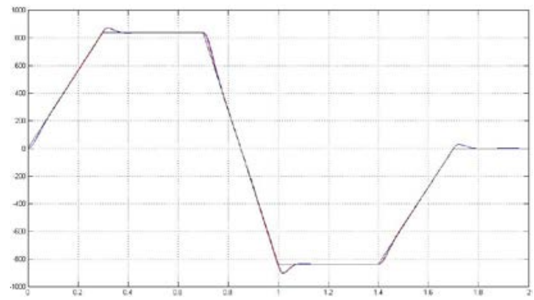

Fig. 8 ω_r real speed compared to the $\hat{\omega}_r$ estimated speed and reference speed: $\omega_r^* = 3000 \cdot \frac{\pi}{30}\left[\frac{\text{rad}}{\text{s}}\right]; M_r = 0$.

5. CONCLUSIONS

This paper presents a new flux and rotor resistance observer called an Extended Gopinath Observer (EGO). The design of the EGO observer is done based on an adaptive mechanism using the notion of Popov hyperstability.

The EGO observer designing using the method presented in this paper ensures the control system with a very good dynamics and robustness. This net avantage, recomend the succesful use of this method in practice.

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