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# EXPERIMENTAL VERIFICATION OF CALCULATION FORMULAS FOR HYDRAULIC CHARACTERISTICS OF ASH SUSPENSION REMOVAL

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**Abstract:** *The article presents a comprehensive analysis of the hydraulic characteristics of ash suspensions, in particular the removal of ash from the Dnieper Hydroelectric Station. The importance of this research is justified by the relevance of understanding hydrodynamic processes in suspensions, which is crucial for the efficiency of mineral resource processing. The study focuses on the investigation of the physico-chemical properties of suspensions, which is fundamental for the development and optimization of technological processes. The main goal of the article is to determine the discrepancies between theoretical calculations and experimental data in the context of the hydraulic characteristics of ash suspensions. Focusing on this issue, the research aims to make an important contribution to the accuracy of prediction and calculation of the behavior of suspensions, which has important practical applications. The research methodology includes a series of experiments to measure various hydraulic parameters of the suspension, including suspension density, solid phase weight content, and porosity. The experiments are based on a detailed analysis of ash samples collected from the Dnieper Hydroelectric Station. It is established that theoretical formulas for determining hydraulic characteristics are well confirmed experimentally. The difference is due to the accuracy of measuring the density of the solid phase and errors in measuring weight and volume; it is sufficiently small and does not have a fundamental character.*
**Keywords:** *ash removal, hydraulic characteristics, density*

## 1. Introduction

The study of methods for the utilization and processing of fly ash from TPPs has significantly engaged researchers due to its rich composition, including carbon and metal oxides, making the ash a valuable resource for use [1-2]. The improvement of processing methods is aimed at creating efficient, combined methods using gravitational enrichment as a cost-effective solution [3-4]. Developing equipment for the effective separation of fine-dispersed ash mixtures with water is an important step in this direction.

The analysis of hydraulic and hydrodynamic processes occurring in any fine-dispersed suspensions, for example, mineral pulps, is carried out based on the hydraulic characteristics of the medium. Experimentally, the characteristics of suspension density ρs, g/cm3, weight content of the solid phase θ, % (further solid percentage), porosity ε, unit - the ratio of the pore volume to the total volume are determined as follows:

$$\rho_c = \frac{P_{total}}{V_{total}} \quad , \quad \theta = \frac{P_m}{P_{total}} \cdot 100 \quad , \quad \varepsilon = \frac{V_w}{V_{total}} \tag{1}$$

The measured quantities are the weight and volume of the solid and liquid phases: Pt, Vt, Vw (Pw), as well as the weight and volume of the total sample Ptotal, Vtotal.

In works [5, 6], a theoretical model is proposed according to which the hydraulic characteristics of the suspension, as well as the particle settling velocity, can be calculated based on the experimental measurement of only one parameter - the density of the suspension, given or measured at a specified density of the solid phase particles ρt. The density index ρs is used in enrichment plants to control the technological process; it is measured by weighing a pulp sample in a liter beaker. In industrial processing, a mixture of minerals is usually subjected to processing, meaning the suspension is not monomineral but polymoneral.

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For a correct description of the flow of mineral pulps, it is necessary to separately establish the density of the solid phase for each fine-dispersed suspension.

The experimentally measured density ρs, along with the experimentally determined density of the solid phase ρt, are included in the theoretical formulas for calculating θ and ε [6].

$$\theta = 100 \cdot \frac{\rho_m}{\rho_c} \cdot \frac{(\rho_c - 1)}{(\rho_m - 1)}, \quad \varepsilon = \frac{\rho_m - \rho_c}{\rho_m - 1} \tag{2}$$

Based on ε, the concentration β of the solid phase and the viscosity of the suspension ν are calculated [6]:

$$\beta = 1 - \varepsilon, \qquad \nu = \nu_0 \exp \frac{2{,}5 \cdot \beta + 0{,}675 \cdot \beta^2}{1 - 0{,}609 \cdot \beta} \tag{3}$$

## 2. Purpose

The aim of the research was to determine the discrepancy between the theoretical and experimental values of hydraulic characteristics using the example of ash carryover from the Dnieper Hydroelectric Power Station. It was assumed that, given the known ρt, the experimentally measured density ρs is true and determines all other hydraulic characteristics.

## 3. Methods

The complex of experiments to determine the hydraulic characteristics of the ash involved initially determining the density of the solid phase ρt for the sample. Then, several water fillings were made for this sample, creating several suspensions of different densities, for each of which the weight and volume were measured. The value of ρt (void volume and Vt) was refined after each filling. Also, based on these measurements, the experimental values of hydraulic characteristics, such as density ρs, weight content of the solid phase θ (further percentage of solids), and porosity ε - the ratio of pore volume to total volume, were calculated and compared with theoretically calculated values.

The primary experiment consisted of adding water in small doses to the dry ash sample until the entire dry mass from top to bottom of the measuring flask was moistened, but a thin film of water was not observed on the surface of the ash. This means that the pores in the ash were completely saturated with water, but the volume of the sample remained unchanged.

4. Results

In the primary experiment, the initial dry sample had a volume of 50 ml and a weight of 52.8 g. It was gradually moistened in 4 steps, with a total amount of water poured to complete wetting being 17+3+2+2=24 ml. The volume of the sample did not change - 50 ml, but the weight increased to 76.8 g (Table 1).

Table 1. Determination of the density of the solid phase of ash (26.09.2023)

| **Weight of ash, g** | **Volume of ash, ml** | **Added water, g or ml** | **Weight of water, g** | **Volume of water, ml** | **Total weight, g** | **Total volume, ml** | **θ, % Rt/ Rw** | **$P_c$, g/cm$^3$ Ptotal/ Vtotal** | **ε, unit, Vw/Vtotal.** |
|---|---|---|---|---|---|---|---|---|---|
| **Rt** | **Vt** | **Δv** | **Pw** | **Vw** | **Rtotal.** | **Vtotal.** | | | |
| 52.8 | 50 | 17+3+2+2 = 24 | 24 | 24 | 76.8 | 50 | 68.75 | 1.536 | 0.48 |
| Solid phase volume: Vt = Vtotal. – Vw = 50 – 24 = 26 cm$^3$<br>Density of the solid phase: ρt = Pt / Vt = 52.8/26 = 2.031 g/cm$^3$. | | | | | | | | | |

Thus, the volume of voids (pores) in the initial dry sample is equal to the volume of added water - 24 cm$^3$. Then the actual volume of the solid phase, without voids: Vt = Vtotal – Vw = 50 – 24 = 26 cm$^3$. The density of the solid phase: ρt = Pt / Vt = 52.8/26 = 2.031 g/cm$^3$.

The initial porosity ε - the ratio of the pore volume to the total volume: ε=V/ Vtotal =24/50=0.48. The initial volumetric concentration of the solid, as the ratio of the volume of the solid phase to the total volume, determined through porosity is: β =1- ε= 26/50=0.52.

The threshold value θ - weight content of the solid phase (further solid percentage) is θ= 100·Pt/ Rtotal = 68.75%, the threshold density ρs=1.536 g/cm$^3$ (table 1).

At the specified threshold values of θ and ρs, the raw material is not a suspension, but only a wet mass in which the movement of particles under the action of gravity does not occur. Only with further dilution does the mixture acquire suspension properties. Therefore, the porosity and solid concentration obtained in table 1 are called initial, and density and solid percentage are called threshold. For this sample, the raw material becomes a suspension when θ and ρs are less than those indicated in table 1.

Several remarks should be made regarding the measurement of ash's solid phase density. The particle size of the ash carried away averaged 250-0 μm. About 80% of it consists of classes smaller than 75 μm, while larger classes of 1-2 mm and up to 4 mm occupy only a small percentage of the total mass. The ash carried away is a polydisperse mixture in which the density of the solid phase depends on the chemical composition of minerals that have undergone high-temperature transformations in thermal power plant boilers along with the burned coal, so ρt requires separate measurements. Such measurements were carried out over the course of a year on samples of ash from the Dniprovska thermal power plant (table 2).

*Table 2. Experimental measurements of the density of the solid phase of ash ρt*

| № | Sample weight, Pt, g | Volume, ml | | | Porosity, ε=Vw/Vtotal, unitless | Density, g/cm³ ρt = Pt/Vt |
|---|---|---|---|---|---|---|
| | | Samples, Vtotal | Voids (water in ash), Vw | Solid phase of ash, Vt | | |
| 1 | 52.8 | 50 | 24 | 26 | 0.48 | 2.031 |
| 2 | 35.89 | 30 | 14.49 | 15.51 | 0.483 | 2.30 |
| 3 | 35.61 | 30 | 11.5 | 18.5 | 0.383 | 1.925 |
| 4 | 10.7 | 10 | 4 | 6 | 0.4 | 1.783 |
| 5 | 208.7 | 190 | 95 | 95 | 0.50 | 2.20 |
| 6 | 30.52 | 28.2 | 12.07 | 16.13 | 0.428 | 1.892 |
| Average | 62.37 | 56.37 | 26.84 | 29.52 | 0.446 | 2.11 |

From Table 2, it follows that ash is a light, fluffy mass in which the pores make up about half of the volume. There is no reliable relationship between ε and ρt. The fluctuations in density ρt can be explained by changes in the chemical composition of individual samples, as well as the presence of closed internal voids and microcracks in particles where water cannot penetrate. Furthermore, in the experiments, for each sample, ρt was first determined separately and then the necessary measurements and calculations were continued.

In the experimental determination of ρt, it is important to carefully establish the actual volume of the solid in the ash, that is, the volume of voids. Secondly, it is necessary to maintain a balance: so that the measured volume and weight of the mixture equal the sum of the corresponding indicators of the liquid and solid phases without voids. The total volume of the mixture Vtotal can be determined when the boundary between ash and water is leveled and accurately measured. The refinement was carried out as follows: the sample was filled with water so that the water stood above the surface. Then the sample was allowed to settle for 3-5 hours, and only the volume of the sediment was measured, which will be the refined volume of the mixture, i.e., the initial dry sample in which the voids are completely filled with water.

The comparison of theoretical formulas for hydraulic characteristics with experimental values will be shown below using a small experiment where only 4 suspensions were created and analyzed, and the results will be presented in tables. The comparison in the form of graphs and correlation dependencies will be shown later in an expanded experiment.

In Table 1, the density of the solid phase of the ash sample is determined as ρt = 52.8/ (50-24) = 2.031 g/cm$^3$. It is noted that the obtained mixture is not a suspension, it is only a moistened ash mass in which particle movement and separation cannot occur. This experiment is denoted by number 1 in Table 3. In further addition of water, the raw material acquires suspension properties. In experiments 2-5, Table 3, to the moist mass obtained from exp. 1, water was added four times in succession. The aim was to create 4 suspensions of different densities. After each addition of water, the weight and volume of the resulting mixture were measured.

From table 3, it can be seen that the more liquid the suspension, the higher the porosity ε. Accordingly, the lower will be the derivatives of porosity - volumetric concentration of the solid phase β=1- ε and viscosity, ν=f(β).

Experiments have shown that the weight of the mixture of ash with water, with some error, equals the sum of the weight of dry ash and the weight of added water (weighing accuracy - 0.02 g). However, experimentally measured volume of the mixture does not equal the sum of the volumes of dry ash and water, it is always smaller. This is explained by the fact that some water is used to fill the gaps (pores) between

particles. When all pores between particles are filled with water, the material becomes wet mass, not a suspension. The mixture becomes a suspension only when the amount of water begins to exceed the amount of filled gaps.

*Table 3. Experimental determination of ρs, θ, ε of ash suspensions (26.09.2023)*

| № | Weight of ash, g | Volume of ash, ml | Added water, g or ml | Weight of water, g | Volume of water, ml | Total | | density, g/cm3 $\rho_C$=Ptotal/Vtotal | Solid fraction θ=100·Pt/Ptotal. | Porosity ε = Vw/Vtotal |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Weight, g | Volume, ml | | | |
| | Pt | Vt | Δv | Pw | Vw | Ptotal | Vtotal | | | |
| 1 | **52.8** | **50** | 17+3+2+2 | 24 | 24 | 76.8 | **50** | 1.536 | 68.75 | 0.48 |
| 2 | 52.8 | 26 | 10.17 | 34.17 | 34.17 | 87.57 | 61 | 1.436 | 60.3 | 0.56 |
| 3 | 52.8 | 26 | 17.25 | 51.42 | 51.42 | 104.51 | 78 | 1.340 | 50.5 | 0.659 |
| 4 | 52.8 | 26 | 17.98 | 69.4 | 69.4 | 121.9 | 95 | 1.283 | 43.3 | 0.731 |
| 5 | 52.8 | 26 | 19.14 | 88.54 | 88.54 | 142.2 | 115 | 1.237 | 37.1 | 0.77 |

* The dry sample of ash has a volume of 50 ml, and the volume of the solid phase in it is 50-24=26 ml (exp.1).

When determining the experimental values using formulas (1), it is necessary to ensure that in each of the experiments No. 1-5, the balance of material flows is maintained with minimal error:

$$P_t + P_w = P_{total}\ ,\ V_t + Vw = V_{total} \quad (4)$$

Special attention should be paid to the fact that in the second equation Vt is the volume of the solid phase without voids. If the balance (4) is not maintained, it is necessary, first of all, to clarify Vt and, accordingly, ρt. The error of experimental determination of ρs, θ, ε depends on the error of the quantities entering formulas (1). For the weight of Pt and Pw (or Vw, because the density of water is 1 g/cm$^3$), high measurement accuracy is ensured by using scales with an accuracy of ±0.02 g. The error in determining Ptotal, Vtotal according to table 3 is given in table 4

*Table 4. Absolute and relative error in determining Ptotal, Vtotal **

| Exp. No. | Ptotal, g | | Error in Ptotal | | Vw, ml | | Error in Vw | |
|---|---|---|---|---|---|---|---|---|
| | Experiment | Calculated from balance (3.3) | Absolute, g | Relative,% | Experiment | Calculated from balance (3.3) | Absolute, ml | .Relative,% |
| 2 | 87.57 | 86.97 | 0.6 | 0.690 | 61 | 60.17 | 0.83 | 1.379 |
| 3 | 104.51 | 104.22 | 0.29 | 0.278 | 78 | 77.42 | 0.58 | 0.749 |
| 4 | 121.9 | 122.20 | 0.3 | 0.245 | 95 | 95.40 | 0.40 | 0.419 |
| 5 | 142.2 | 141.34 | 0.86 | 0.608 | 115 | 114.54 | 0.46 | 0.402 |
| Average | | | 0.51 | 0.46 | Average | | 0.57 | 0.74 |

*Absolute error: |Experimental Ptotal - Theoretical Ptotal |, relative: 100·|Experimental Ptotal - Theoretical Ptotal |/Calculated Ptotal, for Vtotal is similar.

From table 4, it can be seen that the absolute and relative errors for Ptotal, Vtotal are very small, so the experimental values of θ, $\rho_c$, ε in table 3 can be considered reliable based on formulas (1).

In general, the errors in determining the characteristics Xi are defined as:

- absolute error: |Xi experimental - Xi theoretical|;
- relative: 100·|Xi experimental - Xi theoretical|/ Xi true.

When determining the reliability and accuracy of the theoretical formulas, the experimental value was considered as the true value.

A comparison of the experimental values using formulas (1) and the theoretical values using formulas (2), (3) was carried out for the characteristics θ, ε. The experimental values of θ, ε from table 3 were used. The density of the solid phase ρt = 2.031 g/cm$^3$ was also taken from table 3 and used in formulas (2), (3).

The results were evaluated based on absolute and relative error, where the experimental values of θ, ε were considered as true. Thus, an assessment was made of how accurately the theoretical formulas describe the process, i.e., how well they match the experiment (table 5).

*Table 5. Comparison of experimental and theoretical (using formulas 2) values of θ, ε at ρt = 2.031 g/cm³*

| №. | | θ, % | | Error θ | | ε, unit | | Error ε | |
|---|---|---|---|---|---|---|---|---|---|
| | ρc exp. | Exp. | Theor. Formula (3.5) | Absolute, % | Relative, % | Exp. | Theor. Formula (3.6) | Absolute, unit | Relative, % |
| 1 | 1.536 | 68.75 | 68.75 | 0.0 | 0.0 | 0.48 | 0.480 | 0.0 | 0.0 |
| 2 | 1.436 | 60.3 | 59.78 | 0.52 | 0.86 | 0.56 | 0.577 | 0.017 | 3.04 |
| 3 | 1.340 | 50.5 | 49.97 | 0.55 | 1.09 | 0.659 | 0.670 | 0.011 | 1.67 |
| 4 | 1.283 | 43.3 | 43.48 | 0.16 | 0.37 | 0.731 | 0.725 | 0.005 | 0.68 |
| 5 | 1.237 | 37.1 | 37.68 | 0.55 | 1.48 | 0.77 | 0.771 | 0.001 | 0.13 |
| Average | | | | 0.356 | **0.76** | - | | 0.007 | **1.11** |

From the table, it can be seen that the deviations of theoretical values from experimental ones for θ, ε are very small, with the maximum relative error in determining θ not exceeding 1.5% (1.48%) and ε not exceeding 3%, which is acceptable and makes the calculated formulas (2), (3) suitable for practical use.

A similar check for the viscosity index was performed for the measured ρc and ρt. The porosity ε is included in the experimental Wendel formula for viscosity.

$$\nu = \nu_0 \exp\frac{2{,}5 \cdot \beta + 0{,}675 \cdot \beta^2}{1 - 0{,}609 \cdot \beta} \tag{5}$$

Where ν0 = 0.01 cm²/s is the kinematic viscosity of water at 20°C; β = 1- ε is the coefficient of volume concentration of solid in the suspension. The verification of the Wendel formula as such is not part of the research tasks; this formula was objectively chosen by us from a large number of similar ones. The Wendel formula is widely used and has a significant advantage over similar ones because it covers the widest range of densities for aqueous suspensions of various solids.

The task was to verify the coincidence of the kinematic viscosity ν obtained by experimental and theoretical determination of porosity ε (Table 5), taking into account that the concentration β = 1- ε. Using the true experimental value of ε, and correspondingly β and ν were taken. The calculations are presented in Table 6 and shown in Fig.1.

*Table 6. Error in determining viscosity using formula (5) with experimental and theoretical values of ε (β)*

| No. Exp. | $P_c$ Exp. | Concentration, β=1- ε, units | | Viscosity, ν, cm²/s | | Error | |
|---|---|---|---|---|---|---|---|
| | | Experimental | Theoretical | Exp. | Theor. | Absolute, units | Relative,% |
| 1 | 1.536 | 0.520 | 0.520 | 0.087548 | 0.087548 | 0 | 0 |
| 2 | 1.436 | 0.440 | 0.423 | 0.053719 | 0.048898 | 0.004821 | 8.97 |
| 3 | 1.340 | 0.341 | 0.330 | 0.032382 | 0.030787 | 0.001595 | 4.93 |
| 4 | 1.283 | 0.269 | 0.275 | 0.023695 | 0.024281 | 0.00059 | 2.49 |
| 5 | 1.237 | 0.230 | 0.229 | 0.020344 | 0.020267 | 0.00008 | 0.39 |
| Average | | | | | | 0.0014 | 3.35 |

From table 6 it can be seen that in the investigated range of densities, the viscosity according to the experiment is slightly higher than the calculated one, with a maximum relative error of 8.97% corresponding to high density. On average, the relative error of 3.35% is acceptable for the use of the theoretical formula (3).

Thus, for the suspension parameters θ, ε, ν at measured ρs and ρt, sufficient agreement was established between the calculated theoretical data and the experimental ones. All theoretical formulas were tied to one experimentally measured parameter - density. The results showed that the agreement is acceptable, therefore the theoretical formulas are suitable for practical use.

For completeness of the research, let's consider inverse formulas that allow to determine the pulp density by another parameter - the measured percentage of solids or porosity (or volumetric concentration as β = 1- ε).

$$\rho_{c=}\frac{100}{100 - \theta + \theta / p_t} \tag{6}$$

$$\rho_{c=} p_t \cdot (1 - \varepsilon) + \varepsilon \tag{7}$$

The experimental values of ρs, θ, ε are taken from table 3 at ρt = 2.031 g/cm$^3$, and the comparison results are shown in table 7.

*Table 7. Comparison of density according to formulas (6), (7) with experimental*

| № | Experimental values | | | $p_c$, by formula (3.8), g/cm$^3$ | Error in determining ρs | | $\rho_c$, by formula (3.9), g/cm$^3$ | Error in determining $\rho_c$ | |
|---|---|---|---|---|---|---|---|---|---|
| | θ, % | ε, unit | $p_c$, g/cm$^3$ | | Abs., unit | Rel.,% | | Abs., unit | Rel.,% |
| 1 | 68.75 | 0.48 | 1.536 | 1.536 | 0.000 | 0.000 | 1.536 | 0.000 | 0.000 |
| 2 | 60.3 | 0.56 | 1.436 | 1.441 | 0.005 | 0.378 | 1.453 | 0.018 | 1.240 |
| 3 | 68.75 | 0.659 | 1.340 | 1.345 | 0.005 | 0.373 | 1.351 | 0.011 | 0.850 |
| 4 | 60.3 | 0.731 | 1.283 | 1.282 | 0.001 | 0.105 | 1.278 | 0.005 | 0.420 |
| 5 | 50.5 | 0.77 | 1.237 | 1.232 | 0.004 | 0.347 | 1.237 | 0.001 | 0.052 |
| Average | | | | | 0.003 | 0.24 | - | 0.007 | 0.51 |

The maximum relative error in calculating ρs using formula (6) is approximately 0.4%, and using formula (7) with the use of porosity, it reaches 1.24%, which is equal to the absolute difference between the calculated and experimental values of 0.018 g/cm$^3$. Such a deviation can be considered insignificant for practical purposes, meaning that the calculation formulas (6), (7) are acceptable for practical use.

In conclusion, it should be noted that it is theoretically possible to determine the density of the solid phase of a substance based on the experimental indicators of suspension ρs and θ.

$$\rho_{t=}\frac{1}{1-\frac{100\cdot(\rho_c-1)}{\theta\cdot\rho_c}} \tag{8}$$

However, both Pc and θ are experimentally determined with some error, so such a calculation of ρt gives not a unique value, but some fluctuations around the true ρt, which was previously determined experimentally by filling water into voids in the dry mixture of ash and was equal to 2.031 g/cm$^3$ (Table 8).

*Table 8. Comparison of the density of the solid phase ρt according to formula (8) with the experiment*

| No. exp. | $\rho_c$, g/cm3 exp. | θ, exp. | Pt, g/cm$^3$ by formula (3.9) | Error relative to the experimental value ρt = 2.031 g/cm$^3$ | |
|---|---|---|---|---|---|
| | | | | Absolute, g/cm3 | Relative ,% |
| 1 | 1.536 | 68.75 | 2.031 | 0.000 | 0.01 |
| 2 | 1.436 | 60.3 | 2.013 | 0.018 | 0.89 |
| 3 | 1.340 | 68.75 | 2.008 | 0.023 | 1.11 |
| 4 | 1.283 | 60.3 | 2.039 | 0.008 | 0.37 |
| 5 | 1.237 | 50.5 | 2.063 | 0.031 | 1.55 |
| Average | | | 2.031 | 0.016 | 0.79 |

As seen from the table, the maximum relative error of the calculation of ρt reaches 1.55%, which equals the absolute deviation of the calculated value from the experimental value of 0.031 g/cm$^3$. Such a difference is acceptable in practice, but it should be noted that ρt should not be determined based on just one measurement of $\rho_c$ and θ. The more measurements of ρs and θ, the closer the average ρt is to the true value. Considering the difficulty of such determination, it is recommended to directly experimentally determine ρt based on the determination of porosity and the volume of the solid phase, as described above in section table 1.

***Comparison of theoretical and experimental results of determining $\rho_t$, $\rho_c$, θ, ε by an extended experiment***

Let's present a graphical analysis of the experimental and theoretical results of determining the characteristics $\rho_t$, $\rho_s$, θ, ε for the extended experiment in which 10 suspensions created from one sample of ash were analyzed. The data for determining ρt are provided in table 9, and for constructing graphs in table 10.

*Table 9. Determination of the density of the solid phase of ash ρt (03.10.2023)*

| № | Weight of ash, g | Volume of ash, ml | Added water, g or ml | Weight of water, g | Volume of water, ml | Total | | Solid θ = ρc/ ρtotal | Density, g/cm³ ρc = ρtotal/ Vtotal | Porosity, unitless ε = Vwater/Vtotal |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Weight , ml | Volume, мл | | | |
| | Pt | Vt | Δv | Pw | Vw | Ptotal | Vtotal | | | |
| 1 | **35.61** | **30** | 11.5 | 11.5 | 11.5 | 47 | **30** | 75.59 | 1.57 | 0.383 |

The volume of voids in the initial dry ash sample is equal to the volume of water added - 11.5 $cm^3$.

The volume of the solid phase without voids Vt = Vtotal - Vvoids = 30 - 11.5 = 18.5 $cm^3$.

The density of the solid phase: $\rho_t$ = Pt /Vt = 35.61/18.5 = 1.925 $g/cm^3$**.**

Porosity ε, or the void fraction ε = Vvoids/Vtotal = 11.5/30 = 0.383.

Initial volume concentration of the solid β = 1- ε, or β = Vt/Vtotal = 18.5/30 = 0.617.

The critical weight content of the solid phase (further referred to as the solid percentage) θ = 100·Pt/Ptotal = 75.59%. The critical density ρs = Ptotal/ Vtotal = 1.57 $g/cm^3$.

Previous theoretical studies have indicated that the most densely packed particle arrangement corresponds to a porosity ε = 0.259. This value corresponds to the maximum critical density and solid percentage. When ε = 0.259 for particles with ρt = 1.925 $g/cm^3$, the critical values should be:

$p_c$,crit.= $\rho_t$ *0.741+0.259=1.925*0.741+0.259=1.66 ($g/cm^3$)

θcrit=74.1* $\rho_t$ / $\rho_c$,crit =74.1*1.925/1.685= 84.6 (%)

In fact, we have $\rho_c$,crit= 1.57 $g/cm^3$ and θcrit=75.59% (table 9), because the porosity is not 0.259, but higher - ε = 0.383. These are the critical values at which we have a dense moist mass in which particle movement under the action of gravity does not occur.

Thus, in practice, the density of the suspension should be even lower to allow particle separation. Experimental observations confirm this: at a density of about 1.4, it is a dense mass with a small layer of water above it, requiring thorough mixing, after which this mass quickly settles as a solid layer; only at a rare density, about 1.35 and lower, layering of the solid phase is visually observed during sedimentation.

Experimentally, it has been established that for organizing the separation of ash particles during sedimentation, the actual limitation on density and solid percentage is: $\rho_c$ <1.3 - 1.4 $g/cm^3$ and θ < 40 -50%.

Table 10 presents experiments on creating suspensions of different densities, with duplicate experiments on the same ash sample indicated by a dash.

*Table 10. Determination of ρc, θ, ε at ρm =1.925 g/cm³ (continuation of the experiment from Table 9)*

| № | Weight of ash, g | Volume of solid ash, ml | Weight/ volume of water, g/ml | Total | | Experimental | | | Theoretical | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Weight, g | Volume, ml | ρs, g/cm3 | θ, % | ε, unitless | θ, % | ε, unitless |
| | Pt | Vt | Pw/ Vw | Ptotal | Vtotal. | Ptotal/ Vtotal | 00·Pt/ρtotal | Vw/ Vtotal | Formulas (3.5), (3.6) | |
| 2 | 35.61 | 18.5 | 23.72 | 58.98 | 42 | 1.404 | 60.4 | 0.565 | 59.92 | 0.563 |
| 3 | 35.61 | 18.5 | 29.13 | 64.74 | 47 | 1.378 | 55.0 | 0.620 | 57.04 | 0.592 |
| 4 | 35.61 | 18.5 | 35.64 | 69.705 | 52 | 1.340 | 51.1 | 0.685 | 52.86 | 0.632 |
| 5 | 35.61 | 18.5 | 53.56 | 86.88 | 71 | 1.224 | 41.0 | 0.754 | 38.04 | 0.758 |
| 6 | 35.61 | 18.5 | 71.49 | 104.6 | 90 | 1.162 | 34.0 | 0.794 | 29.05 | 0.825 |
| 6' | 35.61 | 18.5 | 71.83 | 106.53 | 91 | 1.171 | 33.4 | 0.789 | 30.34 | 0.815 |
| 7 | 35.61 | 18.5 | 77.25 | 112.14 | 96 | 1.168 | 31.8 | 0.805 | 29.95 | 0.818 |
| 8 | 35.61 | 18.5 | 101.3 | 134.03 | 120 | 1.117 | 26.6 | 0.844 | 21.79 | 0.874 |
| 9 | 35.61 | 18.5 | 97.25 | 132.31 | 117 | 1.131 | 26.9 | 0.831 | 24.08 | 0.859 |
| 10 | 35.61 | 18.5 | 92.25 | 127.08 | 112 | 1.135 | 28.0 | 0.824 | 24.70 | 0.854 |
| 11 | 35.61 | 18.5 | 127.2 | 161.73 | 147 | 1.100 | 22.0 | 0.866 | 18.96 | 0.892 |
| 11' | 35.61 | 18.5 | 127.2 | 162.86 | 146 | 1.115 | 21.9 | 0.872 | 21.55 | 0.875 |
| 12 | 35.61 | 18.5 | 160.8 | 193.12 | 179 | 1.079 | 18.4 | 0.899 | 15.22 | 0.915 |

Figures 1 - 4 show graphs comparing the experimental and calculated values of the hydraulic characteristics for the density of the solid phase of ash ρt = 1.925 $g/cm^3$.

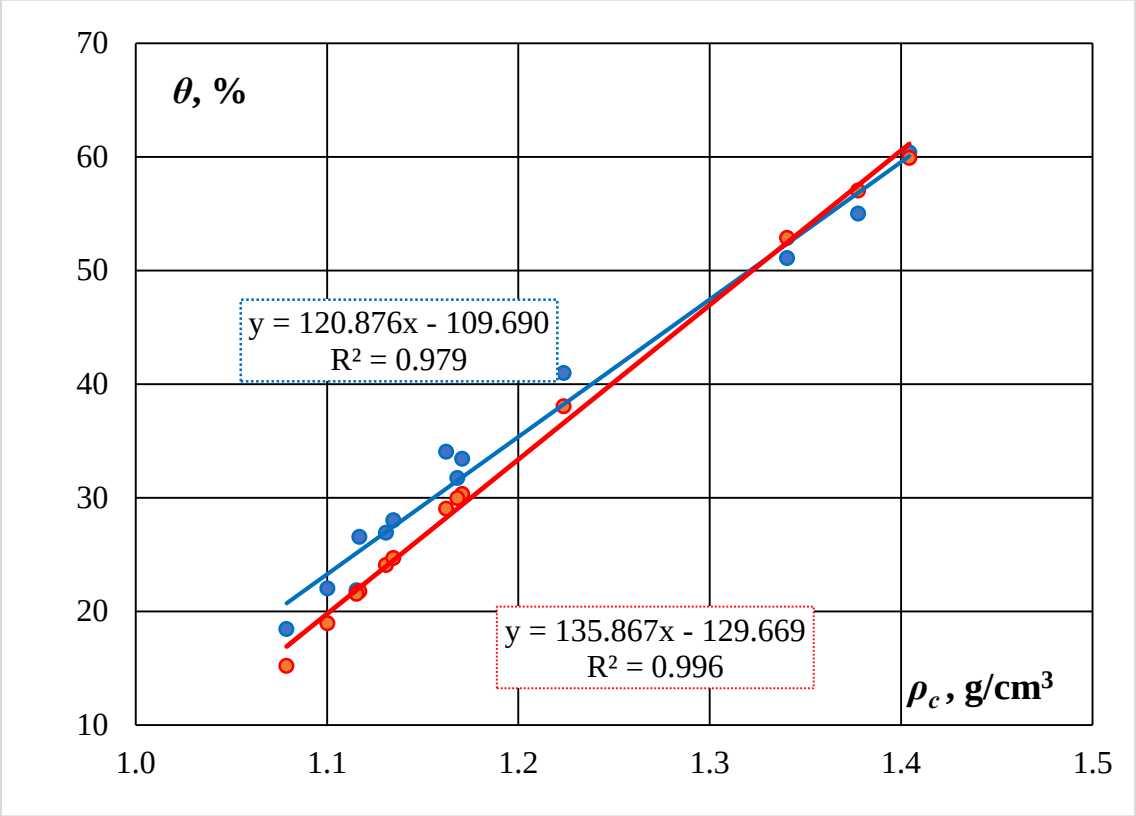


*Fig.1. Dependence of the percentage of the solid phase θ on the density of the suspension ρc, upper - experimental, lower - according to formula (2). Average error of θ: absolute 2.6%, relative 8.4%*

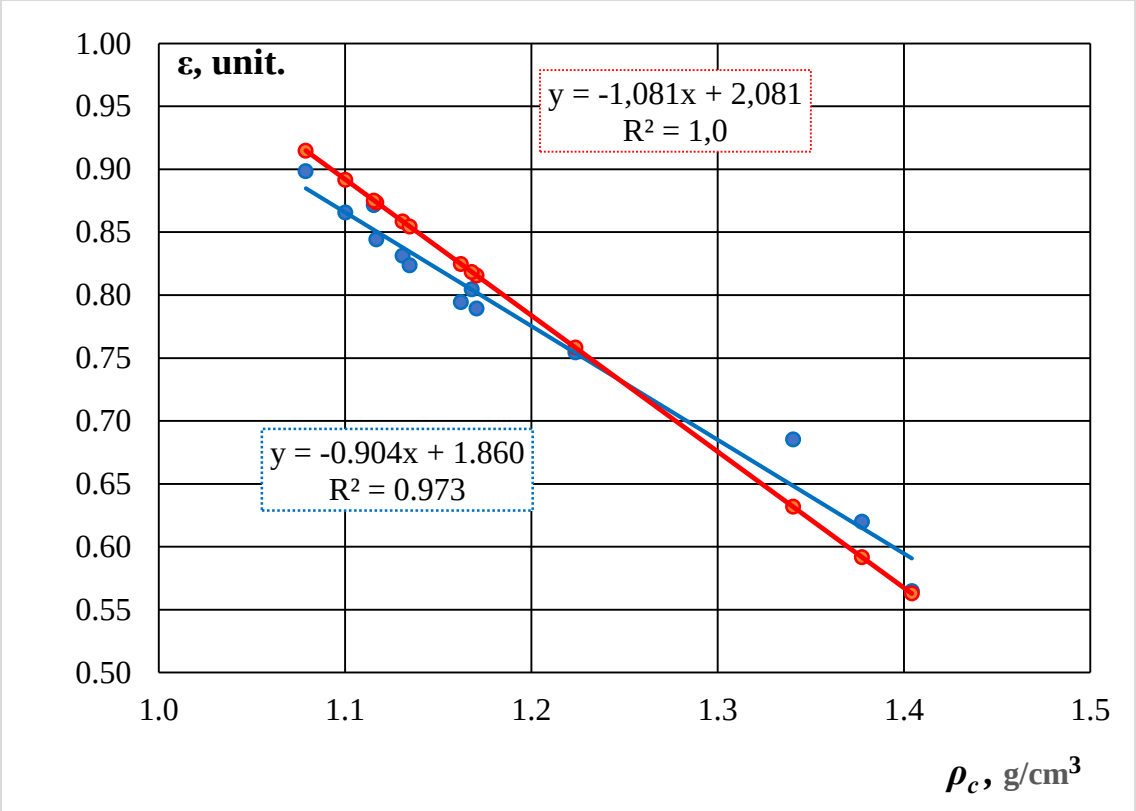


*Fig. 2. Dependence of porosity ε on the density of the suspension ρs, upper - theoretical according to formula (3), lower - experimental. Average error of ε determination: absolute - 0.023 units, relative - 2.99%*

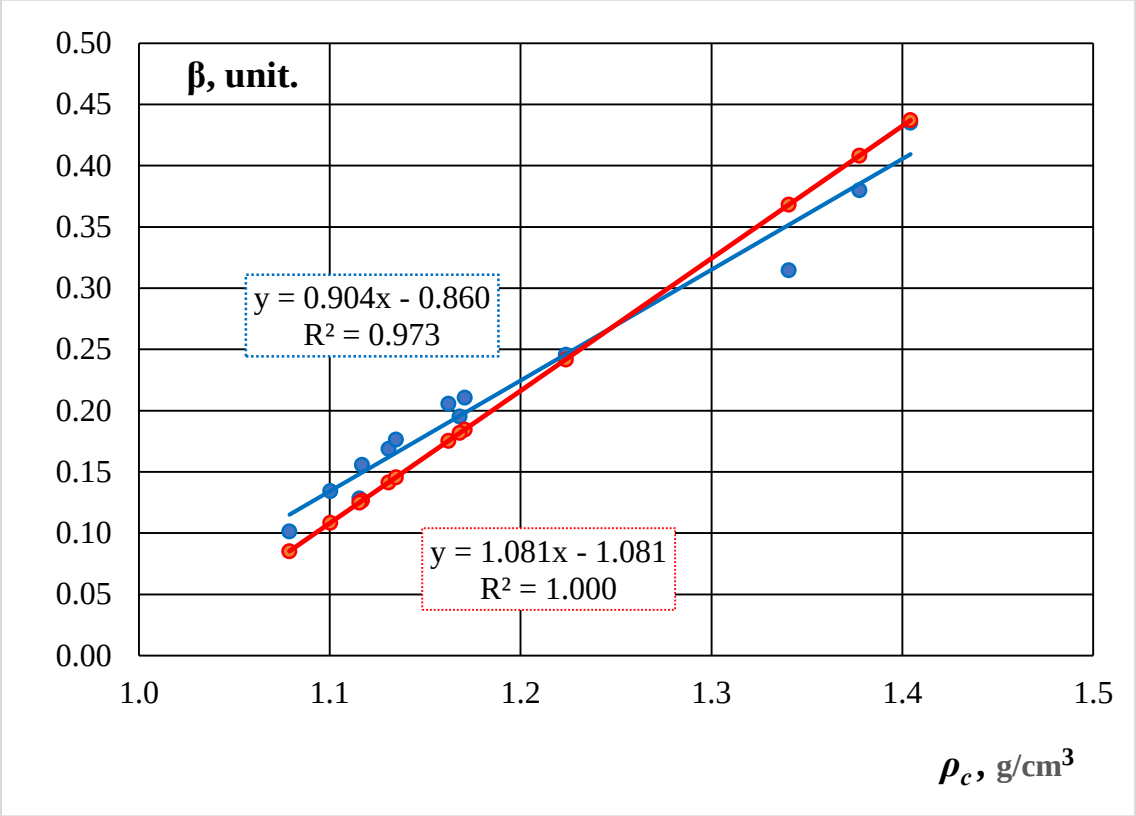


*Fig. 3. Dependence of the concentration of the solid phase β=1 – ε on the density of the suspension ρs, upper - experimental, lower - theoretical, ρt = 1.925 g/cm³. Average error of β determination: absolute 0.023 units, relative 11.2%*

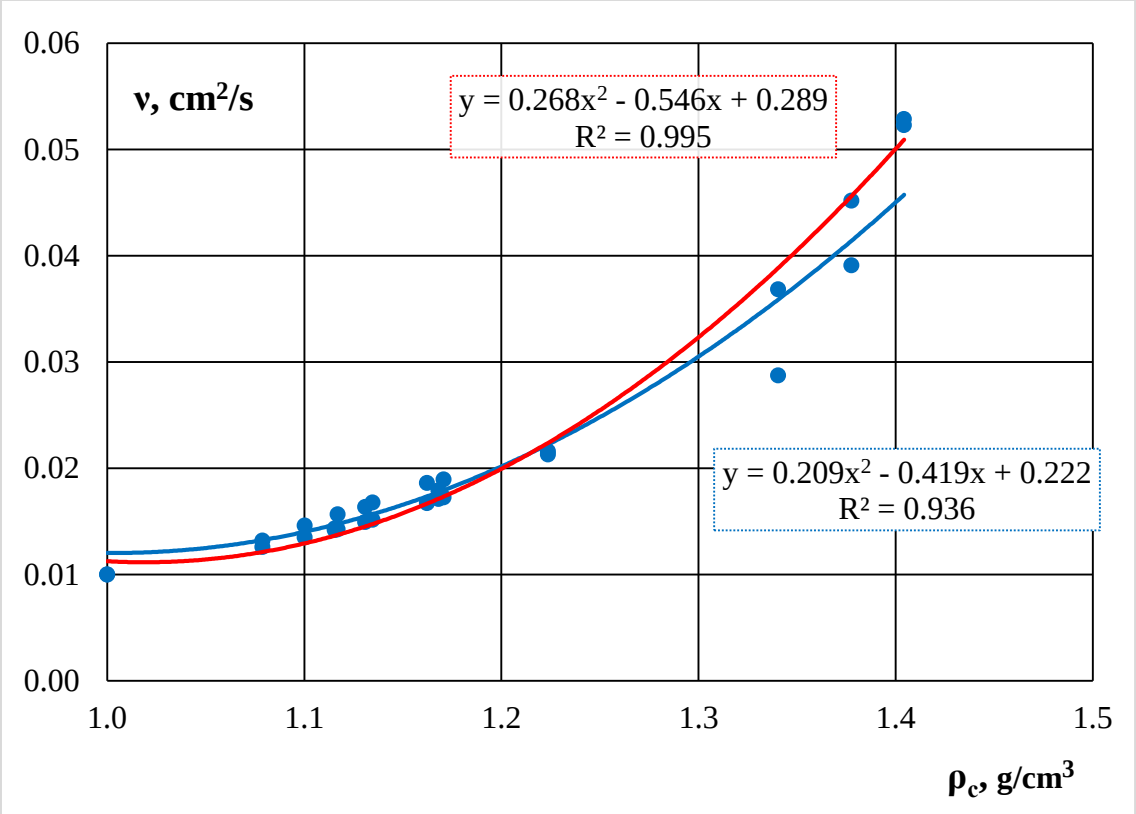


*Fig. 4. Dependence of viscosity on density according to formula (5), where the upper one is experimental (β, ε are experimental), and the lower one is theoretical. Average error: absolute 0.002 cm²/s; relative 8.51%.*

We do not check formulas (6), (7) because these formulas are inverse, obtained from already verified formulas (2), (3), which have shown sufficient agreement between calculated and experimental results for practical purposes.

Graphical dependencies in Figures 1-4 were obtained for $\rho t = 1.925$ g/cm$^3$, covering the density range of 1.404 - 1.079 g/cm$^3$.

Experimental dependencies in Figures 1-4 are approximated by correlation equations:

$\theta = 120.876\rho_c - 109.69$, $R^2 = 0.979$

$\varepsilon = -0.904\rho_c + 1.86$, $R^2 = 0.973$

$\beta = 0.904\rho_c - 0.86$, $R^2 = 0.973$

$\nu = 0.209\rho_c^2 - 0.419\rho s + 0.222$, $R^2 = 0.936$

Using these formulas, experimental indicators for the given density were calculated and compared with calculated values (Table 11).

*Table 11. Hydraulic indicators of ash suspensions of different densities*

| **Theoretical indicators according to formulas (2), (3), (5)** | | | | |
|---|---|---|---|---|
| **$P_c$, g/cm$^3$** | **θ, %** | **ε, unitless** | **β, unitless** | **ν, cm$^2$/s** |
| 1.1 | 18.9 | 0.8919 | 0.108 | 0.013 |
| 1.15 | 27.1 | 0.8378 | 0.162 | 0.016 |
| 1.2 | 34.7 | 0.7838 | 0.216 | 0.019 |
| 1.3 | 48.0 | 0.6756 | 0.324 | 0.030 |
| 1.4 | 59.46 | 0.5675 | 0.432 | 0.052 |
| **Experimental indicators according to correlation formulas based on dependencies in Figures 1-4** | | | | |
| 1.1 | 23,3 | 0,8656 | 0,134 | 0,014 |
| 1.5 | 29.3 | 0.8204 | 0.180 | 0.017 |
| 1.2 | 35.4 | 0.7752 | 0.230 | 0.020 |
| 1,3 | 47.4 | 0.6848 | 0.327 | 0.031 |
| 1.4 | 59.54 | 0.5944 | 0.425 | 0.045 |

It can be seen from Table 11 that for dense suspensions with a density above 1.15-1.2 g/cm$^3$, the difference between experimental and theoretical indicators is insignificant. This indicates that theoretical calculations of suspension characteristics can be used for determining the velocity in the density range of 1.15-1.4 g/cm$^3$.

Summarizing the results, it can be noted that the analysis of dependencies in Figures 1-4 and the data in Table 11 showed that the calculated formulas for determining the characteristics θ, ε, β, ν depending on ρs provide satisfactory agreement with experimental data and can be used in practice, for example, for monitoring the state of pulp in enrichment plants, in chemical processes for processing raw materials, in solving hydraulic and hydrodynamic problems, designing hydraulic equipment, etc.

## 4. Conclusions

The primary characteristic underlying the determination of the characteristics of ash water suspensions is the density of the solid phase. An experimental method was used to determine it, in which the pores in the ash bulk were filled with water and their volume was determined. It was established that for unclassified ash, the pores in the ash occupy approximately half of the volume, on average 47%, and the density of the solid phase can range from 1.78 to 2.3, on average 2.11 g/cm$^3$. Fluctuations in solid density can be explained by non-uniformity of the chemical composition of the ash and the presence of closed voids in the particles. Changes in solid phase density cause changes in hydraulic characteristics, which, in turn, cause changes in particle settling velocity under other equal conditions -with equal particle size and medium density.

It was established that theoretical formulas for determining hydraulic characteristics are well confirmed experimentally. The difference is due to the accuracy of measuring solid phase density and errors in measuring weight and volume; it is quite small and does not have a fundamental character. To confirm this, a graphical comparison of experimental and theoretical results for determining characteristics $\rho_t$, $\rho_c$, θ, ε, β, ν for an extended experiment was presented, in which 10 suspensions of different densities created from one ash sample were analyzed. Thus, theoretical calculations of hydraulic characteristics of ash suspensions, with prior experimental determination of solid phase density, can be recommended for use in engineering calculations of various hydraulic processes.

## References

[1] **Nadutyi V.P., Sevastianov V.S., Kostyria S.V.,** 2016
*Justification of Expediency of Complex Processing of Fly Ash of Thermal Power Plants*. Geotekhníchna Mekhaníka, 131, 59-66

[2] **Ryabov J., Yu.V., Delitsyn L.M. et al.,** 2019
*Methods of Enrichment of Ash and Slag Waste from Coal TPPs and Ways of Their Involvement in Economic Circulation (review).* Teploenergetika [Heat Power Engineering], 3, 3-24

[3] **Shvets V.Y., Rozdobudko E.V., Solomina, G.V.,** 2013
*Aggregated Methodology of Multicriterion Economic and Ecological Examination of the Ecologically Oriented Investment Projects.* Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 3, 139-144

[4] **Sklyar L.V., Prokopchuk D.S.,** 2017
*Technology of Concentration of Slags and Sludge from Thermal Power Plants. Girnichiy Visnik*, 102, 91-96. Access mode: http://nbuv.gov.ua/UJRN/girvi_2017_102_20

[5] **Shevchenko G.O., Cholyshkina O.G., Sukharyev V.V., Kurilov V.S.,** 2023
*Mathematical Model for Determining the Hydraulic Characteristics of Finely Dispersed Water Mineral Suspensions.* Geotekhníchna Mekhaníka, 163, 155-164

[6] **Nadutyi V.P., Kurilov V.S., Cholyshkina O.G., Hankevych V.F.,** 2021
*Analytical Studies of Constrained Particle Settling Speed in a Water Suspension of Fly Ash from Thermal Power Plant* Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu / NGU-Dnipro, 2021.- № 1(181). - p. 32-39.- DOI: 10.33271/nvngu/2021-1/032