Revista Minelor – Mining Revue
ISSN-L 1220-2053 / ISSN 2247-8590
vol. 32, issue 2 / 2026, pp. 78-88

ANALYTICAL EVALUATION OF INTERNAL FORCES IN METAL MINE SUPPORTS UNDER COMBINED VERTICAL AND VARIABLE LATERAL ROCK PRESSURE

Ladislau RADERMACHER[1 *]

[1]University of Petrosani, Romania, ladislauradermacher@upet.ro

DOI: 10.2478/minrv-2026-0015
received –11.05.2026 / revised – 18.05.2026
accepted – 03.06.2026 / published – 19.06.2026

Abstract: *The structural response of metal mine supports is strongly influenced by the interaction between vertical roof pressure and laterally varying rock pressure acting along the support height. This paper presents an analytical evaluation of internal forces in metallic underground reinforcements subjected to combined vertical loading and horizontally varying distributed pressure. The formulation is based on the force method and provides generalized expressions for bending moment, normal force [1, 2, 3], and shear force in any section of the support structure. The horizontal pressure is introduced as a continuous function of the vertical coordinate, allowing realistic representation of rock–support interaction [4, 5]. The horizontal thrust is determined through compatibility conditions, accounting for both bending and axial deformability of the support profile. Dimensional verification and limiting case analysis confirm the mathematical consistency of the derived expressions. The model provides a rigorous analytical tool for assessing stress distribution and optimizing metallic support systems in underground excavations.*
Keywords: *metal supports, rock pressure, lateral thrust, force method, underground structures*

Nomenclature

Symbol	Description	Units
M	Bending moment	kNm
N	Normal (axial) force	kN
T	Shear force	kN
H	Horizontal thrust (reaction)	kN
V	Vertical reaction	kN
p_y	Uniformly distributed vertical load	kN/m
$p_x = f(y)$	Vertically varying horizontal load	kN/m
l	Total span of the gallery	m
h	Height of the vertical column	m
R	Radius of the circular arch	m
φ	Angle measured from the vertical	rad
η	Dummy integration variable along the vertical coordinate	m
$E \cdot I$	Flexural rigidity	kNm²
$E \cdot A$	Axial rigidity	kN
W	Section modulus	m³
A	Cross-sectional area	m²
σ_c	Stress in the material	kN/m²

* Corresponding author: Radermacher Ladislau, assoc.prof. Ph.D. eng., University of Petrosani, Petrosani, Romania, Contact details: University of Petrosani, 20 University Street, ladislauradermacher@upet.ro

1. Introduction

Underground excavations induce significant modifications in the stress state of the surrounding rock mass, generating complex interaction mechanisms between the terrain and the installed support system [4, 5]. The redistribution of stresses following excavation produces both vertical roof loading and lateral pressures acting along the height of the support structure. The accurate evaluation of these effects is essential for ensuring structural safety and long-term stability.

Metal supports composed of vertical columns and curved arches are widely used in mining galleries due to their mechanical efficiency and adaptability. Their structural behavior is

governed not only by the magnitude of the applied loads but also by their spatial distribution. In practice, lateral rock pressure often varies with depth, while vertical roof loading may act simultaneously along the curved arch. These combined actions produce bending moments, axial forces, and shear forces whose accurate assessment requires a consistent analytical framework [2].

Traditional simplified approaches frequently assume idealized loading schemes or neglect the influence of variable horizontal pressure. However, field observations indicate that lateral pressure may vary significantly along the height of the support, depending on geological conditions, excavation sequence, and local rock mass behaviour [5]. Therefore, a more rigorous analytical treatment is required to describe the internal force distribution within the metallic reinforcement.

The present study develops an analytical formulation for metal mine supports subjected to combined vertical loading and vertically varying horizontal pressure. The horizontal load is expressed as a continuous function of the vertical coordinate, enabling the representation of realistic pressure distributions. Using the force method [1, 2, 3], generalized expressions are derived for bending moments, normal forces, and shear forces in both the vertical and curved portions of the support. The compatibility condition associated with horizontal thrust is employed to obtain a closed-form expression for the reaction force [6, 7].

The proposed formulation maintains analytical clarity while preserving mechanical rigor. The resulting expressions allow evaluation of stresses at any section of the support and facilitate structural optimization under specific geotechnical conditions. The consistency of the model is verified through dimensional analysis and examination of limiting configurations.

2. Literature Review

Historical Development of Metal Supports in Underground Mining

The use of metallic reinforcements in underground excavations has a long history, evolving alongside mining engineering practices. Early support systems relied on timber due to its availability and ease of installation, but the increasing depth of mining operations and the corresponding rise in rock pressures necessitated stronger and more durable materials [8]. The transition to metal supports began in the late 19th century, initially using cast iron segments before the widespread adoption of rolled steel profiles in the 20th century [9].

The geometric optimization of support systems has been a recurring theme in mining research. An early contribution in this field, translated from Russian literature, explored the optimization of three-element elliptical supports (KEP), demonstrating that the shape of the support significantly influences its load-bearing capacity [9]. This work established the principle that the support geometry should ideally follow the natural pressure arch developing in the surrounding rock mass.

Medves [8] provides a comprehensive overview of underground support design, covering both theoretical foundations and practical applications. His work emphasizes the importance of considering the interaction between the support structure and the rock mass, a principle that underpins the present study. The book presents classical solutions for various support configurations and serves as a valuable reference for practicing engineers.

Classical Approaches to Stress Analysis in Arches

The analysis of curved structures such as arches has been a fundamental topic in structural mechanics for over a century. The theoretical framework for understanding the behavior of arches under various loading conditions was established by pioneers in elastic stability and structural analysis.

Timoshenko and Gere's monumental work "Theory of Elastic Stability" [2] provides the mathematical foundations for analysing curved members under combined bending and axial compression. Their treatment of the elastic stability of arches remains a cornerstone of structural engineering, offering rigorous methods for determining critical loads and stress distributions. The energy methods presented in this work, including Castigliano's theorem, form the basis for the compatibility equations used in the present formulation.

vol. 32, issue 2 / 2026
pp. 78-88

The application of the force method (also known as the flexibility method) to statically indeterminate structures is thoroughly covered by Ivan et al. [1] in their comprehensive textbook on structural analysis. This method, which involves releasing redundant constraints and imposing compatibility conditions, is particularly well-suited for analysing two-hinged arches such as the one considered in this study. The authors demonstrate how to derive expressions for redundant forces by minimizing the total strain energy of the system.

Hibbeler's widely adopted textbook "Structural Analysis" [3] presents modern approaches to analyzing frames and arches, including detailed explanations of the force method and its application to various structural configurations. The clear presentation of fundamental concepts makes this work particularly valuable for understanding the theoretical underpinnings of the present formulation.

Analytical Methods for Structures under Distributed Loads

The analysis of structures subjected to distributed loads requires specialized techniques that account for the continuous nature of the applied forces. In the context of underground mining, these loads arise from rock pressure acting along the support elements.

Džaparidze's monograph "Analysis of Metal Supports for Underground Excavations" [6], published in Russian, represents a seminal contribution to the field. This work presents comprehensive analytical solutions for various support configurations subjected to rock pressure, including both uniform and non-uniform load distributions. The author's treatment of the interaction between vertical and horizontal loads provides the theoretical foundation for many contemporary design approaches. Džaparidze's influence can be seen in the fundamental equations used in the present study, particularly in the formulation of the compatibility condition for the horizontal thrust.

Elczner's paper "Elements for Calculating Stresses in Metallic Gallery Supports" [7], published in the Romanian journal "Mine, Petrol și Gaze," directly addresses the problem of stress analysis in arched mine supports. This work presents explicit expressions for bending moments, normal forces, and shear forces in circular arches under combined vertical and horizontal loading. Elczner's formulation assumes uniformly distributed loads, which is adequate for many practical situations but may not capture the complexity of real pressure distributions that vary with depth. The present study extends Elczner's approach by allowing the horizontal load to vary arbitrarily with the vertical coordinate, thereby providing a more general and flexible analytical tool.

The derivation of the horizontal thrust through compatibility conditions, as presented in Section 3 of this paper, follows the methodology established by these classical works. The expressions for the partial derivatives of the internal forces with respect to the redundant reaction H, and the subsequent integration over the structure, are direct applications of the principles laid out by Džaparidze [6] and Elczner [7].

Numerical Methods and Their Limitations

With the advent of powerful computers, numerical methods such as the finite element method (FEM) have become increasingly popular for analysing complex underground structures. These methods offer the advantage of handling nonlinear material behaviour, complex geometries, and detailed rock-support interaction.

However, as noted by Kang in his comprehensive review of support technologies for deep coal mines [10], numerical simulations require careful calibration and validation against analytical solutions or experimental data. Kang's review, published in the International Journal of Coal Science & Technology, synthesizes findings from numerous studies on roadway support and highlights the importance of understanding the fundamental mechanics of rock-support interaction. He emphasizes that while numerical methods are powerful, they should be complemented by analytical approaches that provide insight into the underlying physical mechanisms.

Brady and Brown's authoritative text "Rock Mechanics for Underground Mining" [4] presents a comprehensive overview of the interaction between rock masses and support systems. Their treatment of stress redistribution around excavations and the resulting loads on support structures provides essential background for understanding the loading conditions assumed in the present study. The authors emphasize that rock pressure distributions are rarely uniform and often vary significantly along the height of the excavation, justifying the need for formulations that can accommodate variable loads.

Hoek and Brown's classic work "Underground Excavations in Rock" [5] similarly addresses the practical aspects of support design, drawing on extensive field observations and case studies. Their empirical approach to estimating rock loads provides valuable context for the analytical formulation developed in this paper. The authors note that lateral pressures can exhibit considerable variation depending on geological conditions, excavation sequence, and local rock mass behaviour, further reinforcing the motivation for a generalized approach.

vol. 32, issue 2 / 2026
pp. 78-88

Despite their power, numerical methods have certain limitations when applied to routine design. They require specialized software, skilled operators, and significant computational resources. Moreover, they may not provide the same level of insight into the fundamental behaviour of the structure as analytical solutions. As Timoshenko [2] observed, closed-form analytical expressions, even when based on simplified assumptions, offer invaluable understanding of structural behaviour and serve as essential benchmarks for numerical simulations.

Research Gap and Contribution of the Present Study

The review of the existing literature reveals a gap between classical analytical formulations, which typically assume uniformly distributed loads, and the complex loading conditions encountered in practice, where lateral pressures often vary with depth. While numerical methods can handle such complexity, they lack the transparency and immediate applicability of analytical solutions.

The present study addresses this gap by extending the classical formulation of Elczner [7] to accommodate arbitrarily varying horizontal loads. By introducing the horizontal load as a continuous function $p_x = f(y)$ of the vertical coordinate, the proposed model captures the essential features of realistic pressure distributions while maintaining analytical tractability.

The main contributions of this study can be summarized as follows:

1. Generalization of the classical expressions for bending moments, normal forces, and shear forces to account for variable horizontal loads through appropriate integral terms
2. Derivation of a closed-form expression for the horizontal thrust H based on compatibility conditions, including both bending and axial deformations
3. Dimensional verification confirming the consistency of all derived expressions
4. Limiting case analysis demonstrating that the formulation correctly reduces to known solutions for special configurations

The resulting analytical framework provides a rigorous tool for assessing stress distribution in metallic mine supports under realistic loading conditions, facilitating structural optimization and contributing to safer and more economical underground excavations.

3. Mechanical Interaction between Rock Mass and Metal Support

The stress state within the rock mass is significantly altered by the creation of an underground opening [4]. The removal of the excavated material leads to the loss of the initial equilibrium and to a redistribution of stresses around the contour of the gallery. As a consequence, the installed support system is subjected to both vertical pressures originating from the weight of the overlying rock and lateral pressures generated by the tendency of the rock mass to move toward the excavation void [5].

The vertical pressure exerted by the roof can be modelled, in a first approximation, as a uniformly distributed load acting along the arch of the support. In contrast, the lateral pressure frequently varies along the height of the support elements, depending on the in-situ stress distribution, the geomechanical properties of the rock mass, and the excavation conditions [10]. This variation can be represented by a continuous function of the vertical coordinate, denoted as

$$p_x = f(y)$$

The structural response of the metal support results from the interaction between these applied loads and the stiffness characteristics of its elements [8]. Under the action of these pressures, bending moments, axial forces, and shear forces develop in both the vertical members and the curved arch, their distribution being governed by the geometry of the structure and the boundary conditions at the supports.

The correct determination of the horizontal reaction requires the imposition of a compatibility condition associated with the corresponding displacement. This leads naturally to the application of the force method in the mathematical formulation presented in the following section.

4. Stress Modelling in Metal Reinforcements

Based on the presented analysis, it can be considered that a metal reinforcement is subjected to a vertically distributed load p_y and a horizontally distributed load p_x=f(y), as illustrated in Figure 1.

Through analytical resolution using the force method [1, 2, 3], mathematical expressions have been derived for determining the bending moment M, the normal force N, and the shear force T at any section of the arch, regardless of the type of applied load [1, 9].

$$M = M_p + V \cdot x - H \cdot y - M_o$$

vol. 32, issue 2 / 2026
pp. 78-88

$$N = N_p + V \cdot cos\,\varphi + H \cdot \sin\varphi$$

$$T = T_p + V \cdot \sin\varphi - H \cdot \cos\varphi$$

where:
M_p, N_p, and T_p represent the bending moment, normal force and shear force of a statically determined beam under a given load.
x, y denote the coordinates of the analysed section of the arch.

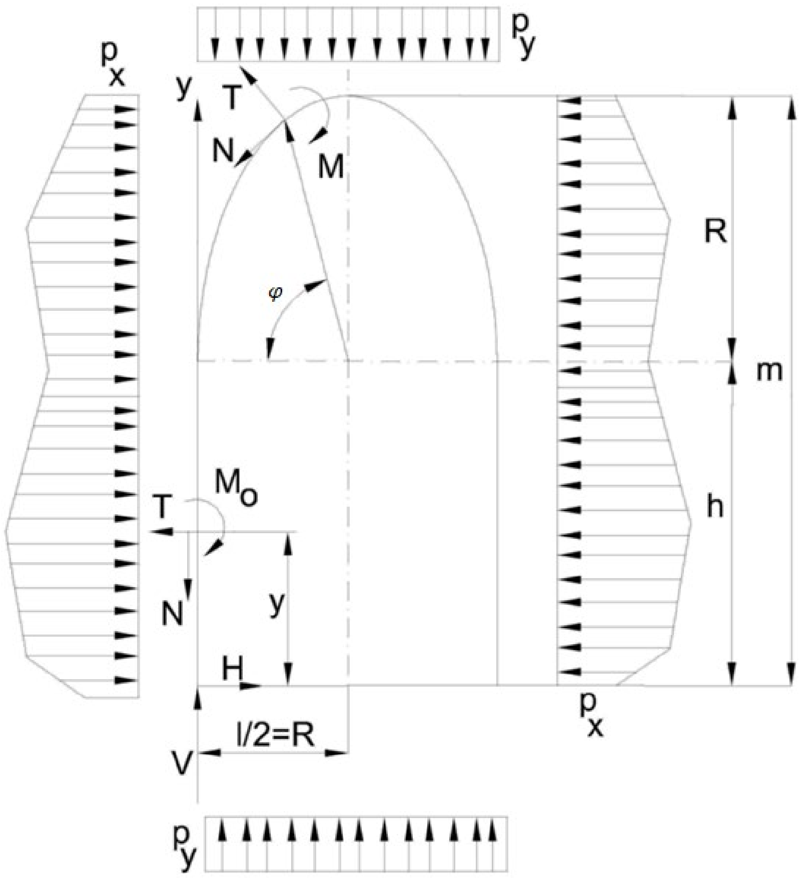

Fig.1. The tasks apply to the metal support

Sign Convention

According to the sign convention shown in Figure 1:

- The vertical load p_y acts downward
- The horizontal load $p_x = f(y)$ acts inward (toward the gallery)
- The horizontal thrust H acts inward (compressing the arch)
- The vertical reaction V acts upward
- The bending moment M is considered positive when it produces tension on the inner face of the arch
- The normal force N is positive when it produces tension
- The shear force T is positive when it produces clockwise rotation on the left section

These coordinates for the arch in Figure 1 are as follows:

$$x = \frac{l}{2} - R \cdot \cos\varphi$$

$$y = h + R \cdot \sin\varphi$$

φ represents the inclination angle of the metallic arch axis at the analysed section, φ =0 for the straight portion.
V, H, M_o are constant values associated with the metallic arch and the type of applied load, where:
V represents the vertical reaction force,
H represents the horizontal reaction force,
M_o represents the bending moment at the base. For a hinged (pinned) support condition, Mo=0.

 vol. 32, issue 2 / 2026
pp. 78-88

If the load from the mine roof (vertical load) p_y is uniformly distributed and the horizontal load is given by $p_x=f(y)$, then the internal forces M, N and T in a given section of the support structure can be determined using the generalized equations: [6, 7, 8]

$$M = V \cdot x - H \cdot y + p_y \cdot \frac{x^2}{2} + \int_0^y f(\eta)(y-\eta)\ d\eta$$

$$T = V \cdot \sin\varphi - H \cdot \cos\varphi + p_y \cdot x \cdot \sin\varphi + \cos\varphi \cdot \int_0^y f(\eta)\ d\eta$$

$$N = -V \cdot \cos\varphi - H \cdot \sin\varphi + p_y \cdot x \cdot \sin\varphi - \cos\varphi \cdot \int_0^y f(\eta)\ d\eta$$

The bending moment at a point with vertical coordinate y is obtained by integrating the contribution of each infinitesimal element. Let η be a temporary variable representing the vertical position of an element, with $0 \le \eta \le y$.

For an element at height η:

- The force acting on this element is $f(\eta)d\eta$
- The lever arm relative to the point at height y is $(y-\eta)$
- The moment contribution is $f(\eta)(y-\eta)d\eta$
- Summing (integrating) over all elements from the bottom ($\eta = 0$) to the point of interest ($\eta = y$) gives:

$$\int_0^y f(\eta)(y-\eta)d\eta$$

Similarly, the total horizontal force acting on the portion from 0 to y is:

$$\int_0^y f(\eta)d\eta$$

In these expressions, η is a "dummy variable" of integration.

For symmetric loading, the vertical reaction at each base is

$$V = \frac{p_y \cdot l}{2}$$

At the hinged (pinned) support point, the generalized displacement conjugate to H is zero. Thus, it can be expressed as follows [6, 7, 8]:

$$\int_0^h \frac{M}{E \cdot I_x} \cdot \frac{\partial M}{\partial H} dy + \int_0^h \frac{N}{E \cdot A} \cdot \frac{\partial N}{\partial H} dy + \int_0^{\frac{\pi}{2}} \frac{M}{E \cdot I_x} \cdot \frac{\partial M}{\partial H} ds + \int_0^{\frac{\pi}{2}} \frac{N}{E \cdot A} \cdot \frac{\partial N}{\partial H} ds$$

where:

$E \cdot I_x$ and $E \cdot A$ represent the bending rigidity and axial rigidity (tension-compression) of the metallic reinforcement profile, respectively.

d_S represents an arc element (a differential length element of the metallic arch).

In general:

d_y refers to a differential element in the vertical portion of the structure (support columns).

d_s refers to a differential element in the curved portion of the metallic arch.

In the analysis of the deformability of a metallic arch, the element $\mathbf{d_s}$ is used to describe variations along the length of the arch, considering that the curved portion cannot be represented solely by the y-coordinate.

In practice, ds can be expressed as a function of the angular coordinate φ as follows:

$$ds = R \cdot d\varphi$$

where:

R is the radius of the arch,

dφ is the angular variation corresponding to an infinitesimal arc element.

Thus, for the lateral column domain, where $0 \le y \le h$, we have: According to the sign convention shown in Figure 1, where both the horizontal thrust H and the lateral rock pressure p_x act inward (compressing the arch), the bending moment in the column is given by [4].

$$M = -H \cdot y - \int_0^y f(\eta)(y-\eta)d\eta$$
$$\frac{\partial M}{\partial H} = -y$$
$$N = -V$$
$$\frac{\partial N}{\partial H} = 0$$

And for the *circular arch portion*, where 0≤φ≤π/2, we have:

$$M = -H \cdot h - H \cdot R \cdot \sin\varphi + V \cdot R \cdot (1-\cos\varphi) - \frac{R^2}{2} \cdot p_y(1-cos\varphi)^2 - \int_0^{h+Rsin\,\varphi} f(\eta)(h + R \cdot sin\phi - \eta)d\eta$$
$$\frac{\partial M}{\partial H} = -(h + R \cdot \sin\varphi)$$
$$N = -V \cdot \cos\varphi - H \cdot sin\,\varphi - p_y \cdot R \cdot (1-cos\varphi) \cdot sin\varphi - \sin\varphi \cdot \int_0^{h+R\sin\varphi} f(\eta)d\eta$$
$$\frac{\partial N}{\partial H} = -\sin\varphi$$

where η is a dummy integration variable along the vertical coordinate, and the coordinates of the arch are given by $x = \frac{l}{2} - R \cdot \cos\ \varphi$ and $y = h + R \cdot \sin\ \varphi$.

Solving the above equation, the lateral thrust H is determined in the following form:

$$H = \frac{T_1 + T_2 + T_3}{\frac{h^3}{3 \cdot E \cdot I_x} + \frac{R \cdot h^2 \cdot \pi}{2 \cdot E \cdot I_x} + \frac{2 \cdot h \cdot R^2}{E \cdot I_x} + \frac{R^3 \cdot \pi}{4 \cdot E \cdot I_x} + \frac{\pi \cdot R}{4 \cdot E \cdot A}}$$

where:

$\frac{h^3}{3 \cdot E \cdot I_x}$ It represents the influence of the bending rigidity of the vertical support column.

$\frac{R \cdot h^2 \cdot \pi}{2 \cdot E \cdot I_x}$ describes the effect of rigidity in the curved section of the arch.

$\frac{R^3 \cdot \pi}{4 \cdot E \cdot I_x}$ represents the influence of rigidity on the arch.

$\frac{\pi \cdot R}{4 \cdot E \cdot A}$ reflects the effect of the normal force on tension and compression.

$$T_1 = \frac{1}{E \cdot I_x} \cdot \int_0^h y \cdot \left[\int_0^y f(\eta)(y-\eta)d\eta\right] dy$$
$$T_2 = \frac{1}{E \cdot I_x} \cdot \int_0^{\pi/2} \left[V \cdot R \cdot (1-\cos\varphi) + \frac{R^2}{2} \cdot p_y \cdot (1-cos\varphi)^2 \cdot \int_0^{h+Rsin\varphi} f\,(\eta) \cdot (h + R \cdot sin\varphi - \eta)d\eta\right] \cdot (h + R \cdot \sin\varphi)d\varphi$$
$$T_3 = \frac{1}{E \cdot A} \cdot \int_0^{\frac{\pi}{2}} \left[V \cdot cos\varphi \cdot sin\varphi + p_y \cdot R \cdot (1-cos\varphi) \cdot sin^2\varphi + sin^2\varphi \cdot \int_0^{\mathrm{h+R\,sin\,\varphi}} \mathrm{f(\eta)\,d\eta}\right] d\varphi$$

5. Dimensional Verification

To ensure the correctness of the derived expressions, a dimensional verification is performed [2, 3]. All terms are dimensionally consistent.

Quantity	Dimensions
$p_y, f(\eta)$	[Force/Length]
$\int f(\eta)d\eta$	[Force/Length] × [Length] = [Force]
$\int f(\eta)(y-\eta)d\eta$	[Force/Length] × [Length²] = [Force·Length]
$E \cdot I$	[Force·Length²]
$E \cdot A$	[Force]
T_1, T_2, T_3	[Length]

vol. 32, issue 2 / 2026
pp. 78-88

Quantity	Dimensions
Denominator	[Length /Force]
$H=(T_1+T_2+T_3)/\text{Denom}$	[Length] / [Length/Force] = [Force]

6. Special cases

Uniform Horizontal Load $f(y)=p_x=$ constant

In this case, the integrals simplify to [6, 7, 8]:

$$\int_0^y f(\eta)d\eta = p_x y$$

$$\int_0^y f(\eta)(y-\eta)d\eta = p_x \frac{y^2}{2}$$

$$\int_0^{h+R\sin\varphi} f(\eta)d\eta = p_x(h+R\sin\varphi)$$

$$\int_0^{h+R\sin\varphi} f(\eta)(h+R\sin\varphi-\eta)d\eta = p_x \frac{(h+R\sin\varphi)^2}{2}$$

The expressions reduce to the classical forms given in [6, 7, 8].

No Horizontal Load $f(y)=0$

In this case, all integrals vanish, and the expressions simplify to the well-known solution for arches under vertical load only [1, 2].

7. Final Equations and Applications

By substituting the values of V and H into the above equations, the relationships for the loads acting on the metallic support system in mining constructions are obtained.

The function $f(y)$ can be determined experimentally by measuring pressure at specific points along the support columns. Through interpolation and mathematical regression, an explicit expression for this function can be obtained [8, 10].

8. Limiting cases

To further validate the derived expressions and to understand the physical behaviour of the support system, it is useful to examine several limiting cases. These limiting cases show how the general formulation reduces to well-known simpler configurations, confirming the consistency of the mathematical model [1, 2].

Vanishing Column Height ($h \to 0$)

When the height of the vertical columns approaches zero ($h \to 0$), the support structure reduces to a circular arch directly hinged at the foundation level [3,4]. In this case, the coordinates of any point on the arch become:

$$x = \frac{l}{2} - R\cos\varphi$$
$$y = R\sin\varphi$$

The expressions for the internal forces in the arch remain valid, but the column contributions disappear. The horizontal thrust H is now determined solely by the arch behaviour. This configuration is typical for shallow underground openings where the arch springs directly from the floor [4].

Very Large Arch Radius ($R \to \infty$)

As the radius of the circular arch becomes very large ($R \to \infty$), the arch flattens and approaches a straight beam configuration. In this limit, the angle φ becomes very small ($\varphi \to 0$), and we have:

$$\sin\varphi \approx \varphi, \cos\varphi \approx 1-\frac{\varphi^2}{2}$$

The coordinates become:

$$x \approx \frac{l}{2} - R\left(1-\frac{\varphi^2}{2}\right) \approx \frac{l}{2} - R + R\frac{\varphi^2}{2}$$
$$y \approx h + R\varphi$$

 vol. 32, issue 2 / 2026
pp. 78-88

For large R, the term $R\varphi$ remains finite (representing the vertical coordinate). When the radius of the circular arch tends to infinity, the curved portion progressively approaches a straight beam configuration. In this limit, the geometry of the structure transforms from an arch into a beam–column system with negligible curvature [1, 2].

From a mechanical standpoint [3]:

- the geometric relation $ds = R\,d\varphi$ implies that curvature effects diminish as $R \to \infty$,
- the bending contribution associated with the arch curvature gradually vanishes,
- the structural behavior approaches that of a straight frame subjected to distributed loads.

Consequently, the analytical expressions derived for the arch tend asymptotically to the classical beam relations [1]. It is important to note that the horizontal thrust Hdoes not necessarily become zero in this limit. Its magnitude depends on boundary conditions and load distribution. Therefore, the limit case $R \to \infty$ should be interpreted as a transition toward straight-beam behavior rather than a complete disappearance of horizontal reaction forces.

Infinitely Rigid Structure ($E \cdot I_x \to \infty$)

If the flexural rigidity of the support profile becomes very large ($E \cdot I_x \to \infty$), the structure approaches an infinitely rigid body. In this case, the deformations become negligible, and the horizontal thrust H can be determined from static equilibrium alone, without considering compatibility conditions.

As $E \cdot I \to \infty$, the flexibility terms tend to zero and the compatibility equation becomes ill-conditioned; H approaches the value obtained from rigid-body equilibrium (limit of the full ratio):

$$H = \frac{T_1 + T_2 + T_3}{\pi/(4 \cdot E \cdot A)}$$

However, since T_1, T_2, and T_3 also contain $1/(E \cdot I_x)$ terms, a more careful limit analysis shows that H approaches the value obtained from rigid body equilibrium. This can be verified by setting all deformations to zero in the original compatibility equation [6, 7].

For the special case of uniform loading ($p_x =$ constant) and neglecting axial deformations ($E \cdot A \to \infty$ as well), the rigid body limit gives:

$$H = \frac{p_y l^2}{8h} \cdot \frac{1 - (p_x/p_y)(h/l)^2}{1 + (h/l)^2}$$

which can be obtained directly from moment equilibrium about the supports [2,8].

These limiting cases demonstrate the consistency of the general formulation and provide useful checks for numerical implementations [10].

Observations

Limiting Case	Behaviour	Verification
$h \to 0$	Arch directly on foundation	General formula → simple arch
$R \to \infty$	Arch → straight beam	General formula → beam
$EI \to \infty$	Infinitely rigid structure	H from static equilibrium

9. Load-Bearing Capacity

Knowing the load distribution and their values, the load-bearing capacity of the beam can be determined, based on the strength condition of the material used in the support structure. This corresponds to the ultimate load, meaning the maximum load that the beam can withstand before the material reaches the plastic deformation domain in a given section of the arch.

The fundamental equation for evaluating the load-bearing capacity is:

$$\sigma_c = \left|\frac{M}{W}\right| + \left|\frac{N}{A}\right| \leq \sigma_y$$

where:

- σ_c represents the computed stress in the material
- σ_y represents the yield strength of the material
- W represents the section modulus of the support profile
- A represents the cross-sectional area of the arch profile
- M and N are the bending moment and the longitudinal force, respectively, acting in the considered section of the beam

vol. 32, issue 2 / 2026
pp. 78-88

Determining the load-bearing capacity allows for the optimization of the support system, ensuring its adaptation to specific underground conditions [10]. Depending on the rock pressures and the geomechanical properties of the surrounding environment, structural modifications to the support system can be implemented to improve stability and safety in mining operations [4, 5].

This approach ensures not only technical efficiency but also optimization of material and economic resources involved in the design and execution of underground works [8].

10. Model Limitations

Although the proposed analytical formulation provides a rigorous evaluation of internal forces in metallic mine supports, several limitations must be acknowledged. The model is developed within the framework of linear elastic structural behaviour and assumes constant mechanical properties (E, I, A) along the support profile. Material nonlinearity, including progressive yielding and plastic redistribution of stresses, is not considered.

Furthermore, the interaction between the rock mass and the support is represented through externally applied distributed loads, without incorporating nonlinear soil–structure interaction mechanisms or contact effects [4, 5]. Time-dependent phenomena such as creep, relaxation, or degradation of rock properties are not included. Dynamic effects induced by blasting operations, seismic disturbances, or impact loading are also outside the scope of the present formulation. Therefore, the model is primarily applicable to elastic analysis under quasi-static loading conditions and should be complemented by advanced numerical methods when nonlinear or dynamic behaviour becomes significant.

11. Conclusions

This paper has presented a comprehensive analytical formulation for evaluating internal forces in metallic mine supports subjected to combined vertical loading and vertically varying horizontal rock pressure. The main findings and contributions of this study are summarized below.

Summary of Contributions

The present study extends classical formulations by introducing the horizontal load as a continuous function of the vertical coordinate, $p_x = f(y)$, thereby capturing the essential features of realistic pressure distributions while maintaining analytical tractability. The generalized expressions derived for bending moments, normal forces, and shear forces reduce to the classical forms found in the literature [6, 7, 8] for the particular case of uniform horizontal load, validating the consistency of the proposed model.

The horizontal thrust H has been determined through a rigorous compatibility condition accounting for both flexural and axial deformations of the support profile [1, 2]. The resulting closed-form expression, together with the detailed terms T_1, T_2, and T_3, provides a complete description of the structural response under arbitrary loading conditions.

Dimensional verification [2, 3] confirms the consistency of all derived expressions, while limiting case analysis demonstrates that the formulation correctly reduces to known solutions for vanishing column height ($h \to 0$), very large arch radius ($R \to \infty$), and infinitely rigid structure ($EI \to \infty$). These checks further validate the mathematical robustness of the model.

Practical Implications

The analytical framework developed in this study offers several practical advantages for the design and optimization of underground support systems:

- **Complete stress determination**: The loads acting on the support system can be evaluated at any point within the structure [1, 2], enabling precise identification of critical sections and informed design decisions [8, 10].
- **Adaptation to geotechnical conditions**: By correlating stress distribution with rock pressure characteristics and the deformable behaviour of the rock mass [4, 5], the model facilitates the selection of technical solutions tailored to specific geological environments.
- **Optimization of resources**: Through careful design based on the derived expressions, greater stability of underground constructions can be achieved while minimizing material consumption and operational costs.

Design Recommendations

Based on the analytical results, the following recommendations are formulated for the practical design of metallic mine supports:

1. **Variable lateral pressure**: For galleries where lateral pressure varies significantly with depth, the classical assumption of uniform pressure may lead to errors of up to 20% in the horizontal thrust H. The general formulation presented in this paper should be employed in such cases to ensure accurate dimensioning.

 vol. 32, issue 2 / 2026
pp. 78-88

2. **Height-to-span ratio**: The ratio h/l has a significant influence on the structural response. For $h/l < 0.5$, the column behavior dominates, and attention should be focused on the column base; for $h/l > 1.0$, the arch behavior becomes prevalent, and the arch crown requires careful verification.
3. **Critical sections**: The load-bearing capacity should be verified at three critical locations: the column base (maximum bending moment), the column-arch junction (transition zone with complex stress distribution), and the arch crown (maximum normal force). The fundamental equation $\sigma_c = | M/W | + | N/A | \leq \sigma_y$ provides a reliable basis for these verifications.
4. **Profile selection**: For metallic supports with moderate slenderness and elastic behaviour [2, 3], the derived expressions enable rational selection of the support profile based on the calculated internal forces, leading to optimal resistance and cost-effectiveness.

Limitations and Future Work

The proposed model is developed within the framework of linear elastic structural behaviour and does not account for material nonlinearity, time-dependent phenomena, or dynamic effects. Future research should focus on experimental validation of the analytical expressions through laboratory tests or field measurements on instrumented supports. Comparison between theoretical predictions and measured strains would provide valuable verification of the model's accuracy. Additionally, extending the formulation to account for nonlinear rock-support interaction and dynamic loading conditions would further enhance its practical applicability.

Concluding Remarks

Unlike classical formulations assuming constant lateral pressure, the present model allows the introduction of a general function $f(y)$, providing increased flexibility for realistic geotechnical conditions [5]. The analytical framework developed in this study offers a reliable basis for the design and optimization of metallic mine supports under complex loading conditions, ultimately contributing to safer and more economical underground excavations [10].

References

[1] **Ivan M., et al.**, 1982
Statistica, stabilitatea și dinamica construcțiilor (in Romanian) Editura Didactică și Pedagogică, București

[2] **Timoshenko S.P., Gere J.M.**, 1985
Theory of Elastic Stability. Second Edition (17th printing). McGraw-Hill Book Company, Inc., New York. Library of Congress Catalog Card Number 59-8568, ISBN 0-07-Y85821-7

[3] **Hibbeler R.C.**, 2016
Structural Analysis. Ninth Edition. Pearson Prentice Hall. ISBN-10: 0-13-394284-8, ISBN-13: 978-0-13-394284-2.

[4] **Brady B.H.G., Brown E.T.**, 2004
Rock Mechanics for Underground Mining. Third Edition. Kluwer Academic Publishers, Dordrecht, The Netherlands. ISBN 1-4020-2046-3

[5] **Hoek E., Brown E.T.**, 2017
Underground Excavations in Rock. Institution of Mining and Metallurgy, London. ISBN 978-1-1384-7392-8

[6] **Džaparidze P.A.**, 1975
Rasčet metaliceskoi krepi gornîh vîrabotok [Analysis of Metal Supports for Underground Excavations]. Nedra, Moscova. (in Russian)

[7] Elczner G., 1987
Elemente de calcul privind eforturile din armăturile metalice ale galeriilor (in Romanian) Revista Mine, Petrol și Gaze, nr. 11, pp. 483-486

[8] **Medves E.R.**, 2018
Calculul susținerii lucrărilor subterane. Teorie și aplicații (in Romanian) Editura A.G.I.R., ISBN 978-973-720-764-7

[9] * * *, 1988
Optimizarea parametrilor geometrici ai susținerii eliptice cu trei elemente (KEP) (in Russian) Șahtnoe stoitelstvo, nr. 10, pp. 11-14.

[10] **Kang H.,** 2014
Support Technologies for Deep and Complex Roadways in Underground Coal Nines: A Review. International Journal of Coal Science & Technology, 1(3), pp. 261-277. https://doi.org/10.1007/s40789-014-0043-0