POSSIBILITIES OF TRANSCALCULATION OF COORDINATES BETWEEN STEREOGRAPHIC AND CENTRAL PROJECTION SYSTEMS

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Abstract: *Geodesy and topography works are performed using the stereographic projection system with secant plane. In this system a point located on the topographic surface is represented by a point. However, there are vertical engineering works (mining, hydro-technics, transport) that connect the surface with the underground and whose axis is presented by two points. The representation of these works by a single point is possible if central projection is used. A study of these projections is required because they are qualitatively different.*

1. Introduction

The stereographic and central projections are part of the group of horizontal azimuthal perspective projections, projections that are suitable in very good conditions for the representation in plan of the surfaces with circular perimeter in the case of small works such as cities, industrial works, etc. So are the mining basins that have themselves, a vertical extension. But the two projections have different qualities. The stereographic projection is compliant and with small linear distortions, the central projection is arbitrary.

On the other hand, the stereographic projection being compliant, there is the possibility of trans-calculating the coordinates between different conformal representation systems. And yet, for the mentioned advantage, the central projection in the mining topographic activity is used. As a result, the following analyses are performed below.

2. Transcalculation of coordinates from the central projection to the stereographic projection.

The deduction of the trans-calculation relations is done owing to the following methods:

- Analytical
- Geometric

2.1. Analytical deduction of trans-calculation formulas

The general expressions (figure 1) of the perspective horizontal projection are:

$$x = \frac{KR\sin v\cos u}{D + R\cos v}$$
$$y = \frac{KR\sin v\sin u}{D + R\cos v} \qquad (1)$$

where:

- K – the distance from the point of view to the center of the projection C
- R – average radius of the Gaussian sphere $R = \sqrt{MN}$
- D – distance from the center of the sphere to the center of the projection
- u - the angle in the projection plane that the direction from the center of the projection makes to a certain point P with the x-axis
- v – the angle at the center of the sphere corresponding to the PC spring

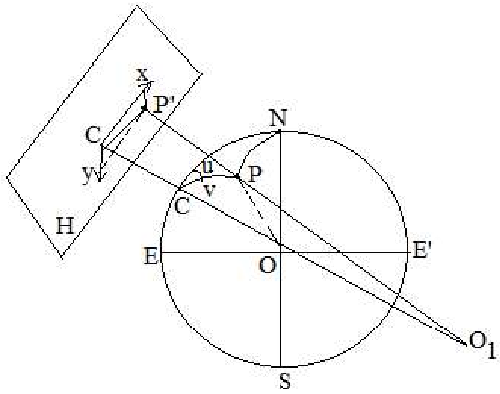

Fig. 1

Equalities (1) correspond to the projections that are trans-calculated.

In the central projection for which:

$$D = 0;\ K = R$$

the relations are:

$$x_c = \frac{R\sin v\cos u}{\cos v}$$
$$y_c = \frac{R\sin v\sin u}{\cos v} \qquad (2)$$

In the stereographic projection for which:

$$D = R;\ K = 2R$$

The relations are:

$$x_s = \frac{2R\sin v\cos u}{1 + \cos v}$$
$$y_s = \frac{2R\sin v\sin u}{1 + \cos v} \qquad (3)$$

Relations (2) and (3) can be written in the form:

$$R\sin v\cos u = x_c\cos v$$
$$R\sin v\sin u = y_c\cos v \qquad (2')$$

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$$R \sin v \cos u = \frac{x_S}{2}(1 + \cos v)$$
$$R \sin v \sin u = \frac{y_S}{2}(1 + \cos v) \quad (3')$$

Equating the groups of equations (2′) and (3′) results:

$$x_c \cos v = \frac{x_S}{2}(1 + \cos v)$$
$$y_c \cos v = \frac{y_S}{2}(1 + \cos v) \quad (4)$$

or:

$$x_c = \frac{x_S}{2} \cdot \frac{1 + \cos v}{\cos v}$$
$$y_c = \frac{y_S}{2} \cdot \frac{1 + \cos v}{\cos v} \quad (4')$$

We note:

$$\frac{1 + \cos v}{\cos v} = K$$

and

$$K_S = \frac{K}{2} \quad (5)$$

then:

$$x_C = K_S x_S$$
$$y_C = K_S y_S \quad (6)$$

But if:

$$K_C = \frac{2}{K} \quad (7)$$

We get:

$$x_S = K_C x_C$$
$$y_S = K_C y_C \quad (8)$$

We notice that the relations (5) and (7) can be:

$$K = 2K_S = \frac{2}{K_C} \quad (9)$$

or:

$$K_S = \frac{1}{K_C} \quad (9')$$

2.2. Geometric deduction of trans-calculation formulas

Consider the figure (2) according to which we can write:

$$\frac{\rho_S}{2R} = tg\frac{v}{2} \quad (10)$$

or:

$$R = \frac{\rho_S}{2tg\frac{v}{2}} \quad (10')$$

and

$$\frac{\rho_C}{R} = tgv \quad (11)$$

or

$$R = \frac{\rho_C}{tgv} \quad (11')$$

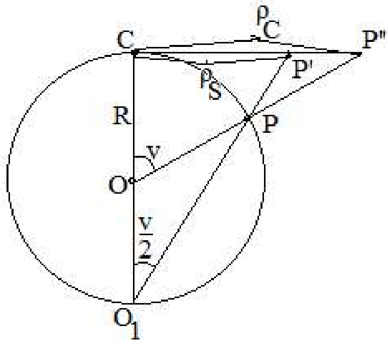

Fig. 2

Equating (10′) with (11′) we get:

$$\rho_C = \rho_S \frac{tgv}{2tg\frac{v}{2}} \quad (12)$$

We know that:

$$tgv = \frac{2tg\frac{v}{2}}{1 - tg^2\frac{v}{2}}$$

then:

$$\rho_C = \rho_S \frac{1}{1 - tg^2\frac{v}{2}} = \rho_S K_S \quad (13)$$

By reversing the relation (12) we obtain:

$$\rho_S = \rho_C\left(1 - tg^2\frac{v}{2}\right) = \rho_C K_C \quad (14)$$

It obviously turns out that:

$$K_S = \frac{1}{K_C} \quad (15)$$

2.3. Calculation of transcalculation constants

a) for central projection (K_C)
it turned out that:

$$K = \frac{1 + \cos v}{\cos v}$$

sau:

$$K = \frac{1}{\cos v} + 1 = \sec v + 1 \quad (16)$$

but:

$$\sec v = \frac{1}{\cos v} = \frac{\sqrt{\rho_C^2 + R^2}}{R}$$

respectively:

$$\sec v = \sqrt{1 + \frac{\rho_C^2}{R}} \quad (17)$$

In the projection plane:

$$\rho_C^2 = x_0^2 + y_0^2$$

and:

$$\sec v = \sqrt{1 + \frac{x_C^2 + y_C^2}{R^2}} \quad (18)$$

The relation (16) becomes:

$$K = 1 + \sqrt{1 + \frac{x_C^2 + y_C^2}{R^2}} \tag{19}$$

With its value K is obtained:

$$K_C = \frac{2}{K}$$

or:

$$K_C = \frac{2}{1 + \sqrt{1 + \frac{x_C^2 + y_C^2}{R^2}}} \tag{20}$$

with:

$$x_S = x_C \frac{2}{1 + \sqrt{1 + \frac{x_C^2 + y_C^2}{R^2}}}$$

$$y_S = y_C \frac{2}{1 + \sqrt{1 + \frac{x_C^2 + y_C^2}{R^2}}} \tag{21}$$

b) For stereographic projection(K_S)

It has been shown that the general relation of the constant is:

$$K = \frac{1 + \cos v}{\cos v} = 1 + \frac{1}{\cos v}$$

but:

$$\cos v = 2cos^2 \frac{v}{2} - 1 \tag{22}$$

and by the figure:

$$\cos \frac{v}{2} = \frac{2R}{\sqrt{\rho_S^2 + (2R)^2}} \tag{23}$$

With (23), (22) is:

$$\cos v = \frac{8R^2}{\rho_S^2 + 4R^2} - 1 \tag{24}$$

and

$$K = 1 + \frac{1}{\frac{8R^2}{\rho_S^2 + 4R^2} - 1} \tag{25}$$

or:

$$K = 1 + \frac{\rho_S^2 + 4R^2}{4R^2 - \rho_S^2} = \frac{8R^2}{4R^2 - \rho_S^2} \tag{26}$$

Finally:

$$K = \frac{2}{1 - \frac{\rho_S^2}{4R^2}} \tag{27}$$

and:

$$K_S = \frac{K}{2}$$

but:

$$\rho_S^2 = x_S^2 + y_S^2$$

with:

$$K_S = \frac{1}{1 - \frac{x_S^2 + y_S^2}{4R^2}} \tag{28}$$

To transcribe the stereographic coordinates into central coordinates, we use the relationships:

$$x_c = \frac{1}{1 - \frac{x_S^2 + y_S^2}{4R^2}} x_S$$

$$y_c = \frac{1}{1 - \frac{x_S^2 + y_S^2}{4R^2}} y_S \tag{29}$$

3. Conclusions

To trans-calculate the coordinates from one projection to another, proceed as follows:

- K_C or K_S is calculated, depending on the nature of the trans-calculation;
- using the established constant, the trans-calculation program is performed

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