SPATIAL POSITIONING
OF A MINIMAL LENGHT UNDERGROUND WORK

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Abstract
Very often it is required the optimal positioning of an underground work, so that it ensures the shortest way connection between two other underground works having certain directions and tilts (galleries, tunnels, winzes, shafts, inclined works). It is already known that positioning can be solved as an intersection of several planes. The current work presents an other way of solving the problem, namely, as an optimization problem by determining an extreme (min or max) and having subject to constraints. This work can also be a model or an example of optimization for solving a problem, with applications in other fields as well.

1. General considerations

It is known from specialty literature [1], [2] that positioning of the shortest connection (way) between two linear underground works having different orientations and inclinations can be done by solving a system of equations written for several planes such as work planes containing axes of the database works or perpendicular planes to the database works. Intersections of these families of planes lead to searched solution. The known model can be applied to straight (linear) works only and with great difficulty at curved works. The present work proposes an other way to approach the problem, treating it as an optimization problem where the aim is to achieve an extreme while respecting certain conditions. This approach can be applied easily not only for linear works but curved works as well, or in situations which require even more technical conditions [3].

2. Formulation of the problem

Let's consider two known underground works with different orientations and inclinations (tilts).

Each work is known by the coordinates of two points (Fig.1), knowing the points $A(x_A\ y_A z_A)$, $B(x_B\ y_B z_B)$, $C(x_C\ y_C z_C)$, $D(x_D\ y_D z_D)$.

The two works do not intersect and are not located on the same plane.

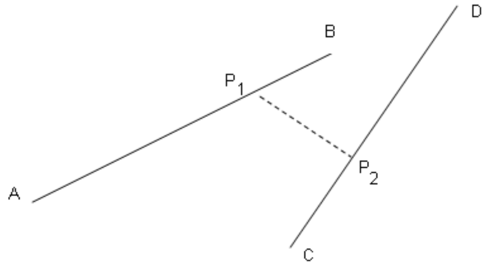

Fig.1

It is required to find the shortest link between the two works, so $P_1(x_1\ y_1 z_1)$ and $P_2(x_2\ y_2 z_2)$ must be determined so that the distance between them is minimal.

3. How we solve

In this case, the function of which extreme must be sought is the „distance" function. Actually, we take into account the square of the distance, without extracting the square root of distance, to avoid negative values (which can be obtained by extracting the square root) – unaccepted solution. The extreme function which we are seeking is:

$$f(x_1 y_1 z_1 x_2 y_2 z_2) = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \quad (1)$$

The conditions which must be taken into account in the search of the minimum, are those which express that the point P_1 must belong to line AB and P_2 must belong to line CD.

For point P_1 can be written these conditions:

$$\frac{x_1 - x_A}{x_B - x_A} = \frac{y_1 - y_A}{y_B - y_A}$$
$$\frac{x_1 - x_A}{x_B - x_A} = \frac{z_1 - z_A}{z_B - z_A} \quad (2)$$

For point P_2 can be written these conditions:

$$\frac{x_2 - x_C}{x_D - x_C} = \frac{y_2 - y_C}{y_D - y_C}$$
$$\frac{x_2 - x_C}{x_D - x_C} = \frac{z_2 - z_C}{z_D - z_C} \quad (3)$$

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The next step is the composition of function F that contains the Lagrange multipliers (there is an analogy between this function and the formation of function for which we are seekeing an extreme in the adjustement process, using least squares direct measurement subject to conditions)

$$F(x_1 y_1 z_1 x_2 y_2 z_2) = f(z_1 y_1 z_1 x_2 y_2 z_2) + \tag{4}$$
$$+2\lambda_1 c_1 + 2\lambda_2 c_2 + 2\lambda_3 c_3 + 2\lambda_4 c_4$$

where c_1 c_2 c_3 c_4 are expressions of the imposed conditions (written in a way to be equal whith zero)

$$c_1 = \frac{x_1 - x_A}{x_B - x_A} - \frac{y_1 - y_A}{y_B - y_A} = 0$$
$$c_2 = \frac{x_1 - x_A}{x_B - x_A} - \frac{z_1 - z_A}{z_B - z_A} = 0$$
$$\tag{5}$$
$$c_3 = \frac{x_2 - x_C}{x_D - x_C} - \frac{y_2 - y_C}{y_D - y_C} = 0$$
$$c_4 = \frac{x_2 - x_C}{x_D - x_C} - \frac{z_2 - z_C}{z_D - z_C} = 0$$

The extreme function F is obtained for those values $x_1 y_1 z_1$ $x_2 y_2 z_2$ that satisfy the conditions:

$$\frac{\partial F}{\partial x_1} = 0 \ , \frac{\partial F}{\partial y_1} = 0 \ , \ \frac{\partial F}{\partial z_1} = 0 \ , \ldots , \frac{\partial F}{\partial z_2} = 0 \tag{6}$$

After the derivation and simplification of the relation (4) we obtain:

$$x_1 - x_2 + \lambda_1 \frac{1}{x_B - x_A} + \lambda_2 \frac{1}{x_B - x_A} = 0 \tag{7}$$
$$y_1 - y_2 - \lambda_1 \frac{1}{y_B - y_A} = 0 \tag{8}$$
$$z_1 - z_2 - \lambda_2 \frac{1}{z_B - z_A} = 0 \tag{9}$$
$$-x_1 + x_2 + \lambda_3 \frac{1}{x_D - x_C} + \lambda_4 \frac{1}{x_D - x_C} = 0 \tag{10}$$
$$-y_1 + y_2 + \lambda_3 \frac{1}{x_D - x_C} = 0 \tag{11}$$
$$-z_1 + z_2 + \lambda_4 \frac{1}{z_D - z_C} = 0 \tag{12}$$

The constants λ_1 ,λ_2 are determined from the equations (8) and (9) and substituted in the equation (7). Also, the constants λ_3 ,λ_4 are determined from the equations (11) and (12) and substituted in the equation (10).

We obtain:

$$(x_1 - x_2) + (y_1 - y_2)\frac{y_B - y_A}{x_B - x_A} + (z_1 - z_2)\frac{z_B - z_A}{x_B - x_A} = 0$$
$$-(x_1 - x_2) - (y_1 - y_2)\frac{y_D - y_C}{x_D - x_C} + (z_1 - z_2)\frac{z_D - z_C}{x_D - x_C} = 0 \tag{13}$$

We make the following notations:

$$tg\theta_{AB} = \frac{y_B - y_A}{x_B - x_A} \qquad tg\theta_{CD} = \frac{y_D - y_C}{x_D - x_C}$$
$$tg\gamma_{AB} = \frac{z_B - z_A}{x_B - x_A} \qquad tg\gamma_{CD} = \frac{z_D - z_C}{z_B - z_A} \tag{14}$$

If we attach the equations (5) multiplied one by one with $(y_B - y_A)$, $(z_B - z_A)$, $(y_D - y_C)$, $(z_D - z_C)$ to equation (13) then we can form the following system:

$$(x_1 - x_2) + (y_1 - y_2)\cdot tg\theta_{AB} + (z_1 - z_2)\cdot tg\gamma_{AB} = 0$$
$$-(x_1 - x_2) - (y_1 - y_2)\cdot tg\theta_{CD} - (z_1 - z_2)\cdot tg\gamma_{CD} = 0$$
$$x_1 \cdot tg\theta_{AB} - y_1 - x_A \cdot tg\theta_{AB} + y_A = 0$$
$$x_1 \cdot tg\gamma_{AB} - z_1 - x_A \cdot tg\gamma_{AB} + z_A = 0$$
$$x_2 \cdot tg\theta_{CD} - y_1 - x_C \cdot tg\theta_{CD} + y_C = 0$$
$$x_2 \cdot tg\gamma_{CD} - z_2 - x_C \cdot tg\gamma_{CD} + z_c = 0 \tag{15}$$

Values $x_1 y_1 z_1$ $x_2 y_2 z_2$, for which the connection is minimal, are obtained by solving the system. The system can be written in a form of a matrix, such as:

$$Ax - l = 0 \tag{16}$$

The system has the solution

$$x = A^{-1} l \tag{17}$$

and the matrix A is:

$$A = \begin{pmatrix} 1 & -1 & +tg\theta_{AB} & -tg\theta_{AB} & +tg\gamma_{AB} & -tg\gamma_{AB} \\ -1 & 1 & -tg\theta_{CD} & +tg\theta_{CD} & -tg\gamma_{CD} & +tg\gamma_{CD} \\ tg\theta_{AB} & 0 & -1 & 0 & 0 & 0 \\ tg\gamma_{AB} & 0 & 0 & 0 & -1 & 0 \\ 0 & tg\theta_{CD} & 0 & -1 & 0 & 0 \\ 0 & tg\gamma_{CD} & 0 & 0 & 0 & -1 \end{pmatrix}$$

Matrices l and x will have the form:

$$l = \begin{pmatrix} 0 \\ 0 \\ x_A tg\theta_{AB} - y_A \\ x_A tg\gamma_{AB} - z_A \\ x_C tg\theta_{CD} - y_C \\ x_C tg\gamma_{CD} - z_C \end{pmatrix} \qquad x = \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \\ z_1 \\ z_2 \end{pmatrix}$$

4. Case study

For a particular case from the Jiu Valley mine, we know two underground works (inclined planes) AB and CD, which are known by these given coordinates:

Point	x	y	z
A	439665.592	375871.259	151.250
B	439806.235	376033.297	133.221
C	439660.498	376050.380	101.550
D	439792.767	376106.945	123.125

These values were obtained in processing:

Matrix *A*

1	-1	1.152123	-1.152123	-0.128190	0.128190
-1	1	-0.427651	0.427651	-0.163115	0.163115
1.152123	0	-1	0	0	0
-0.128190	0	0	0	-1	0
0	0.4276512	0	-1	0	0
0	0.1631146	0	0	0	-1

Matrix *l*

0
0
130677.47
-56511.90
-188029.02
71613.47

Matrix *X*

X1	439901.643
X2	439895.322
Y1	376143.218
Y2	376150.803
Z1	120.991
Z2	139.853

5. Conclusions

Through the establishment of a link of a minimum length respecting the given technical limitations, we can reach a minimum time of digging and a minimum volume and a lower cost of the excavations. The presented model can be adapted to determine the shortest connection between two curved works as well. In this case, changes are in relationships (2) and (3) which are to be written as curve defining expressions.

References

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