ERROR PROPAGATION OF THE SIZES MEASURED IN MINE WORKING

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Abstract

The execution of mine working (vertical, horizontal and inclined) needs measurements and topographic processing caracterized by high accuracy.

It is, therefore, of major importance to determine the degree of accuracy for different situations expressed by general and specific connect functions.

1. Type of connect functions

In topographic mine working, in many cases one or more sizes can't be accessible to direct meassurement, being imposed to be determined from other sizes that are measured directly

The characteristic of this meassurement is that the size or the sizes that have to be determined can be determined by connect functions such as:

a) Linear and explicit
b) Nonlinear and explicit
c) Nonlinear and implicit

These situations can be illustrated as follows:

a) Underground level transmission by roulette on a vertical shaft

In principle, the method consists in determining of the guide length between horizons. For this purpose it is used an ordinary roulette with a length of 50 m.

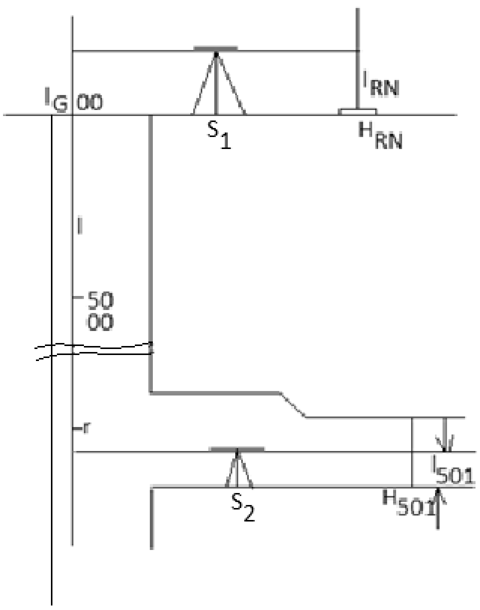

Fig. 1

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Practically a bucket is mounted above the cage at about 50 m high on the rope. On the cage and on the bucket 2 operators will work simultaneously protected by safety belts.

The bucket operator will handle the zero point of the roulette and the other will handle the roulette by the fork. The cage is lowered with 50 m each time, intervals between which on the guides are measured equal portions of 50 m

The measurements performed on the shaft are connected with the measurements performed on the surface and in the underground with 2 stationary levels in points S_1 and S_2 (fig. 1).

These measurements consist in the observations on the ruler of measurement situated in the marks R_N and 501.

Following the figure we can write:

$$H_{501} = H_{RN} + l_{RN} - l_G - (n \cdot 50 + r) - l_{501} + \\ +C \qquad (1)$$

where C is the total correction consisted of: calibration correction, stretching correction and temperature correction.

Thus, the point 501 can't be directly measured but it can be obtained by a linear and explicit function. Consequently there are functions of whose general form can be written:

$$F = C_O + C_1 l_1 + C_2 l_2 + \dots + C_n l_n \qquad (2)$$

where:

$C_O, C_1, C_2, \dots, C_n$ are constants

$l_1, l_2, \dots, l_n$ are the sizes directly measured

b) Level transmission on an inclined plane by the trigonometric leveling method

It is applied at mine working with high inclination (inclined plane) or very high inclination (chute, winze) (fig. 2).

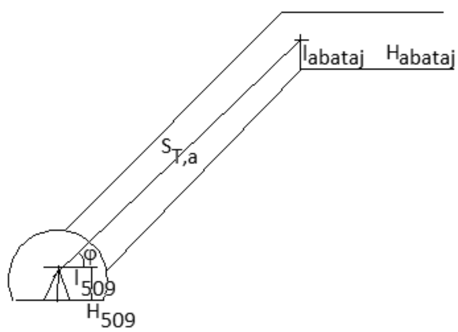

Fig. 2

Basically with the theodolite telescope attached horizontally the ruler of measurement situated on the mark of leveling 509 is aimed and the reading l_{509} .is recorded. The distance on inclination $S_{T,\ a}$, the angle of inclination φ and the reading l_a are measured directly on the ruler of measurement situated in coal face.

The level H_a at low level of coal face results in:

$$H_a = H_{E09} + l_{E09} + S_{T,a}\sin\varphi - l_a \quad (3)$$

If:

$$l_{E09} = l_a \qquad H_a = H_{E09} + S_{T,a}\sin\varphi \quad (4)$$

In this case the connect function between the directly measured sizes and the size that is determined is explicit but nonlinear. As general form we can write:

$$F = f(l_1, l_2, \ldots, l_n) \quad (5)$$

c) Underground junction of topographic elements

At topographic elements junction in the kip of vertical shaft the sides a, b, c and the angle γ are measured (the linking triangle method). (fig. 3).

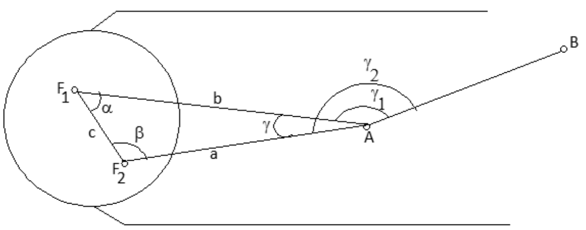

Fig. 3

For the achievement of junction the angles α and β are determined with the relations:

$$\sin\alpha = \frac{a}{c}\sin\gamma \qquad \sin\alpha - \frac{a}{c}\sin\gamma = 0$$

or (6)

$$\sin\beta = \frac{b}{c}\sin\gamma \qquad \sin\beta - \frac{b}{c}\sin\gamma = 0$$

It can be observed that 2 nonlinear and implicit functions were obtained which can be written as:

$$F_1(\alpha, \beta, a, b, c, \gamma) = 0$$
$$F_2(\alpha, \beta, a, b, c, \gamma) = 0 \quad (7)$$

The general form of such functions is:

$$F_1(x_1, x_2, \ldots, x_s, l_1, l_2, \ldots, l_n) = 0$$
$$F_2(x_1, x_2, \ldots, x_s, l_1, l_2, \ldots, l_n) = 0$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$
$$F_s(x_1, x_2, \ldots, x_s, l_1, l_2, \ldots, l_n) = 0$$

for which $s < n$

2. The linearization of functions

The scope of nonlinear explicit and implicit functions linearization is to be able to determine the errors of the sizes that can not be directly measured because they are not accessible. The functions (5) and (8) should be considered.

In function (5), the sizes directly measured $l_1, l_2, \ldots, l_n$ are affected by errors that can be assimilated with the $\Delta l_1, \Delta l_2, \ldots, \Delta l_n$ growths. As result of these growths the function will have the growth ΔF .

So we can write:

$$F + \Delta F = f(l_1 + \Delta l_1, l_2 + \Delta l_2, \ldots, l_n + \Delta l_n) \quad (9)$$

Developing in series we obtain:

$$F + \Delta F = f(l_1, l_2, l_3, \ldots, l_n) + \frac{\partial f}{\partial l_1}\Delta l_1 + \frac{\partial f}{\partial l_2}\Delta l_2 + \cdots + \frac{\partial f}{\partial l_n}\Delta l_n$$

from where:

$$\Delta F = \frac{\partial f}{\partial l_1}\Delta l_1 + \frac{\partial f}{\partial l_2}\Delta l_2 + \cdots + \frac{\partial f}{\partial l_n}\Delta l_n \quad (10)$$

The equality (10) represents the total differential of function (5).

Similarly we can also linearize the functions from equalities (8).

3. The error expression of a function

a) The case of linear function

By differentiation of function (2) is obtained:

$$\Delta F = C_1\Delta l_1 + C_2\Delta l_2 + \cdots + C_n\Delta l_n \quad (11)$$

By transition from differential to errors results:

$$m_F^2 = C_1^2 m_1^2 + C_2^2 m_2^2 + \cdots + C_n^2 m_n^2$$

or:

$m_F = \sqrt{[C^2 m^2]}$; $\quad m_1, m_2, \ldots, m_n$ are mean-square errors of the measured sized (12) (12)

b) The case of nonlinear and explicit function

It was shown that nonlinear function can be linearized, the final form being represented by equality (10).

By transition from growth to errors is obtained:

$$m_F = \sqrt{\left[\left(\frac{\partial f}{\partial l}\right)^2 m^2\right]} \quad (13)$$

c) The case of nonlinear implicit functions

Nonlinear implicit functions represent the general case of functions of sized directly measured, represented by system (8).

The functions exist only when:

$$\frac{D(F_1, F_2, \ldots, F_s)}{D(x_1, x_2, \ldots, x_s)} = \begin{vmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_s} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdots & \frac{\partial F_2}{\partial x_s} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial F_s}{\partial x_1} & \frac{\partial F_s}{\partial x_2} & \cdots & \frac{\partial F_s}{\partial x_s} \end{vmatrix} \neq 0 \quad (14)$$

In order to determine the errors of the functions one can proceed to the linearization of them.

For simplification we note:

$$\frac{D(f)}{D(x)} = \frac{D(F_1, F_2, \dots, F_s)}{D(x_1, x_2, \dots, x_s)} \tag{15}$$

$$\frac{D(f)}{D(x_j)i} = \frac{D(F_1, F_2, \dots, F_s)}{D(x_1, x_2, \dots, x_{j-1}, l_i, x_{j+1}, \dots, x_s)} =$$

$$= \begin{vmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_{j-1}} & \frac{\partial F_1}{\partial l_i} & \frac{\partial F_1}{\partial x_{j+1}} & \cdots & \frac{\partial F_1}{\partial x_s} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial F_s}{\partial x_1} & \cdots & \frac{\partial F_s}{\partial x_{j-1}} & \frac{\partial F_s}{\partial l_i} & \frac{\partial F_s}{\partial x_{j+1}} & \cdots & \frac{\partial F_s}{\partial x_s} \end{vmatrix}$$

$$i = 1, 2, \dots, n$$
$$j = 1, 2, \dots, j \tag{16}$$

The determinant (16) is obtained from the determinant (14) by replacement of the columns of the coefficients of the growths $\Delta x_1, \Delta x_2, \dots, \Delta x_s$ with the absolute terms.

By ration between (16) and 15) the coefficients of the growth result $\Delta l_1, \Delta l_2, \dots, \Delta l_n$ noted with $\frac{\partial x_j}{\partial l_i}$ and it is obtained:

$$\frac{\partial x_j}{\partial l_i} = \frac{\frac{D(F)}{D(x_j)i}}{\frac{D(F)}{D(x)}} \tag{17}$$

If:

$$C = \frac{1}{\frac{D(F)}{D(x)}}$$

$$\frac{\partial x_j}{\partial l_i} = C \frac{D(F)}{D(x_j)i} \tag{18}$$

the mean errors of the functions will be:

$$m_{x_j} = C \sqrt{\left[\left(\frac{DF}{D(x_j)}\right)^2 m^2\right]} \tag{19}$$

4. Example

We consider the general case shown in figure 3. The equalities (6) are differentiated and it is obtained:

$$\cos\alpha\Delta\alpha - \frac{\sin\gamma}{c}\Delta a + \frac{a\sin\gamma}{c^2}\Delta c - \frac{a}{c}\cos\gamma\Delta\gamma = 0$$

$$\cos\beta\Delta\beta - \frac{\sin\gamma}{c}\Delta b + \frac{b\sin\gamma}{c^2}\Delta c - \frac{b}{c}\cos\gamma\Delta\gamma = 0$$

The main determinant is:

$$D = \begin{vmatrix} \cos\alpha & 0 \\ 0 & \cos\beta \end{vmatrix} \neq 0$$

$$\Delta\alpha = \frac{\begin{vmatrix} \frac{\sin\gamma}{c}\Delta a - \frac{a}{c^2}\sin\gamma\Delta c + \frac{a}{c}\cos\gamma\Delta\gamma & 0 \\ 0 & \cos\beta \end{vmatrix}}{\begin{vmatrix} \cos\alpha & 0 \\ 0 & \cos\beta \end{vmatrix}}$$

or:

$$\Delta\alpha = \frac{\sin\gamma}{\cos\alpha}\frac{\Delta a}{c} - \frac{\sin\gamma}{\cos\alpha}\frac{a}{c^2}\Delta c + \frac{\cos\gamma}{\cos\alpha}\frac{a}{c}\Delta\gamma$$

Considering the equalities:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\Delta\alpha = tg\alpha\frac{\Delta a}{a} + tg\alpha\frac{\Delta c}{c} + tg\alpha\frac{\Delta\gamma}{tg\gamma}$$

Proceeding to the error:

$$m_\alpha = tg\alpha\sqrt{\left(\frac{m_a}{a}\right)^2 + \left(\frac{m_c}{c}\right)^2 + \left(\frac{m_\gamma}{tg\gamma}\right)^2}$$

By replacing the second column of the determinant from the denominator with the absolute terms one can obtain:

$$\Delta\beta = \frac{\begin{vmatrix} \cos\alpha & 0 \\ 0 & \frac{\sin\gamma}{c}\Delta b - \frac{b}{c^2}\sin\gamma\Delta c + \frac{b}{c}\cos\gamma\Delta\gamma \end{vmatrix}}{\begin{vmatrix} \cos\alpha & 0 \\ 0 & \cos\beta \end{vmatrix}}$$

$$\Lambda\beta = tg\beta\left(\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta\gamma}{tg\gamma}\right)$$

and

$$m_\beta = tg\beta\sqrt{\left(\frac{m_b}{b}\right)^2 + \left(\frac{m_c}{c}\right)^2 + \left(\frac{m_\gamma}{tg\gamma}\right)^2}$$

Observation:

The growths $\Delta\alpha$ *and* $\Delta\beta$, and also the errors m_α *and* m_β are minim for:

$$\alpha \to 0 \qquad \beta \to 200$$

or

$$\alpha \to 200 \qquad \beta \to 0$$

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