POSITIONAL SYNTHESIS OF THE SLIDER-CRANK INVERTED MECHANISM

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Abstract In the paper we present the positional synthesis of the slider-crank inverted mechanism as part of mining machines and equipment.

Introduction

In figure 1 is shown the slider crank inverted mechanism. The dimension *e* represents the eccentricity of the crank guideway. The contour vectorial equation, in relative dimensions, can be written with the notations in the figure by referring to the AD dimensions.

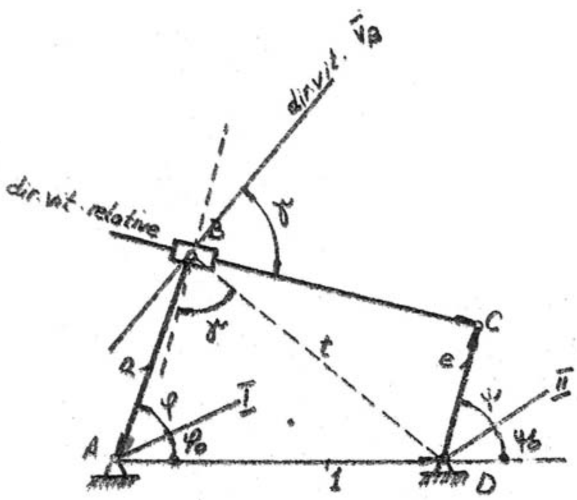

Fig. 1 The slider crank inverted mechanism and the geometrical parameters that define it (a,e,φ_0,ψ_0)

$$BC = \bar{e} + \bar{1} + \bar{a} \quad (1)$$

By squaring and taking into account that:

$$(BC)^2 + e^2 = t^2 = a^2 + 1 - 2a\cos(\varphi_0 + \varphi) \quad (2)$$

we obtain:

$$e - a\cos(\psi_0 + \psi - \varphi_0 - \varphi) + \cos(\psi_0 + \psi) = 0 \quad (3)$$

From equation (3) it results that the rotating output element is dependent on four geometric parameters: *a, e,* φ_0 and ψ_0.

The calculus of three parameters

Let us suppose that we want to determine the parameters *a, e* and φ_0 (the ψ_0 parameter being chosen arbitrarily). In this case the equation (3) is expanded under the form:

$$\begin{aligned} &e - a\cos(\psi + \psi_0 - \varphi)\cos\varphi_0 - \\ &- a\sin(\psi + \psi_0 - \varphi)\sin\varphi_0 + \cos(\psi + \psi_0) = 0 \end{aligned} \quad (4)$$

and then it is identified with the following interpolation polynomial:

$$p_0 + f_0(\varphi) + p_1 f_1(\varphi) + p_2 f_2(\varphi) = 0 \quad (5)$$

where:

$$\begin{cases} p_0 = \dfrac{e}{a\cos\varphi_0} \\ p_1 = \dfrac{1}{a\cos\varphi_0} \\ p_2 = tg\varphi_0 \end{cases} \quad (6)$$

$$\begin{cases} f_0 = -\cos(\psi + \psi_0 - \varphi) \\ f_1 = \cos(\psi + \psi_0) \\ f_2 = -\sin(\psi + \psi_0 - \varphi) \end{cases} \quad (7)$$

In order to determine the three coefficients p_0, p_1 and p_2, the values of the functions $f_0(\varphi)$, $f_1(\varphi)$ and $f_2(\varphi)$ will be calculated for a set of values of angle φ (φ_1, φ_2 and φ_3) in the approximation interval (φ_0,φ_m) chosen arbitrarily or by Chebyshev spacing. A system of three linear equation in p_j, $j=0,1,2$ is obtained:

$$\begin{cases} p_0 + f_0(\varphi_1) + p_1 f_1(\varphi_1) + p_2 f_2(\varphi_1) = 0 \\ p_0 + f_0(\varphi_2) + p_1 f_1(\varphi_2) + p_2 f_2(\varphi_2) = 0 \\ p_0 + f_0(\varphi_3) + p_1 f_1(\varphi_3) + p_2 f_2(\varphi_3) = 0 \end{cases} \quad (8)$$

After finding the parameters p_0, p_1 and p_2 with formulae (6), the mechanism unknown parameters are determined, in the following sequence:

$$\varphi_0 = arctg(p_2);\ a = \frac{e}{p_1\cos\varphi_0};\ e = ap_0\cos\varphi_0 \quad (9)$$

Error analysis

The real value of angle $\psi = \psi_r$ can be calculated with one of the following relations:

$$\psi_r + \psi_0 = arctg\frac{\sin(\varphi_0 + \varphi)}{\cos(\varphi_0 + \varphi) - \frac{1}{a}} \pm \arccos\frac{e}{t} \quad (10)$$

where

$$t = \sqrt{a^2 + 1 - 2a\cos(\varphi + \varphi_0)} \quad (11)$$

or with the relation deducted from the position equation (3), written under the form of equation (12) in $\sin(\psi_0+\psi)$ and $\cos(\psi_0+\psi)$:

$$A\sin(\psi_0 + \psi) + B\cos(\psi_0 + \psi) + C = 0 \quad (12)$$

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whereby it results

$$\psi_0 + \psi_i = 2arctg\left(\frac{A_i \pm \sqrt{A_i^2 + B_i^2 - C_i^2}}{B_i - C_i}\right) \quad (13)$$

where

$$\begin{cases} A_i = a\sin(\varphi_0 + \varphi_i) \\ B_i = -a\cos(\varphi_0 + \varphi_i) + 1 \\ C_i = e \end{cases} \quad (14)$$

The position deviation of the *Δψ(φ)* crank guideway is compared to the allowable value for several values φ_i in the approximation interval (φ_0, φ_m):

$$\Delta\psi(\varphi) = \psi_r - \psi \le \Delta\psi_{ad} \quad (15)$$

The calculation of four parameters

The position equation (3) is written under the form of the following polynomial in order to calculate all the four parameters:

$$p_0 + f_0(\varphi) + p_1 f_1(\varphi) + p_2 f_2(\varphi) + + p_3 f_3(\varphi) = 0 \quad (16)$$

where:

$$\begin{cases} p_0 = \dfrac{a}{\cos\varphi_0} \\ p_1 = a(tg\psi_0\cos\varphi_0 - \sin\varphi_0) \\ p_2 = a(tg\psi_0\cos\varphi_0 + \cos\varphi_0) \\ p_1 = tg\psi_0 \end{cases} \quad (17)$$

$$\begin{cases} f_0 = \cos\psi \\ f_1 = -\sin(\psi - \varphi) \\ f_{21} = -\cos(\psi - \varphi) \\ f_3 = -\sin\psi \end{cases} \quad (18)$$

The abscissas of the interpolation nodes within the approximation interval (φ_0,φ_m) are chosen arbitrarily or with the Chebyshev spacing. The system of four linear equations in p_j, $j=0,1,2,3$ is written and solved.

The unknown parameters of the mechanism are determined in the following sequence:

$$\begin{cases} tg\psi_0 = p_3 \\ e = p_0\cos\psi_0 \\ tg(\varphi_0 - \psi_0) = \dfrac{p_1}{p_2} \\ a = \dfrac{p_2}{tg\psi_0\sin\varphi_0 + \cos\varphi_0} \end{cases} \quad (19)$$

Supplementary conditions

Conditions regarding the elements length

In the case of the mechanism with rotating slotted link, the following relations have to be fulfilled regarding the elements length:

$$\begin{cases} a \le L; \quad e \ge \dfrac{1}{L} \\ a \ge \dfrac{1}{L}; \quad a \le eL \\ e \le L; \quad a \ge \dfrac{e}{L} \end{cases} \quad (20)$$

The transmission angle

In the case when element AB is the driving one, the transmission angle is the angle γ, made between the directions of the absolute velocity of point B (perpendicular on t) and the direction of the relative velocity (parallel to the sliding direction of the slipper).

Rotating the sides of this angle clockwise, we obtain the angle made between a perpendicular direction on the sliding direction BC and the direction BD..

From Fig it results:

$$\cos\gamma = \frac{e}{t} \quad (21)$$

Taking into account relation (11), we get:

$$\cos\gamma = \frac{e}{\sqrt{a^2 + 1 - 2a\cos(\varphi + \varphi_0)}} \quad (22)$$

From relation (22) we deduce that $\gamma = \gamma_{min}$ for $\varphi + \varphi_0 = 0$:

$$\cos\gamma_{\min} = \frac{e}{|a-1|} \quad (23)$$

Therefore, the condition of the transmission angle can be given by relation:

$$|a-1|\cos\gamma_{ad} \ge e \quad (24)$$

From figure 1 it results that slotted link rotates if:

$$a > 1 + e \quad (25)$$

From the inequalities (24) and (25) it can be inferred that for $a > 1$ and by fulfilling the condition (15) the mechanism with rotating slotted link is obtained. In this case, of the inequalities (20) only the following must be fulfilled:

$$\begin{cases} a \le L \\ a \le eL \\ e \ge \dfrac{1}{L} \end{cases} \quad (26)$$

If

$$a < 1 + e \quad (27)$$

by rotating the crank AB, the slotted link will have an oscillating motion.

In order to satisfy condition (24) it is necessary to comply with the following inequality:

$$a \le 1 - \frac{e}{\cos\gamma_{ad}} \qquad (28)$$

which comprises also relation (20).

From the inequalities (20) the following must be fulfilled:

$$\begin{cases} a \ge \frac{1}{L};\ a \le eL \\ e \ge \frac{1}{L};\ a \ge \frac{e}{L} \end{cases} \qquad (29)$$

The synthesis of the crank guideway mechanism

In figure 2is shown the two crank guideways mechanism and its geometrical parameters.

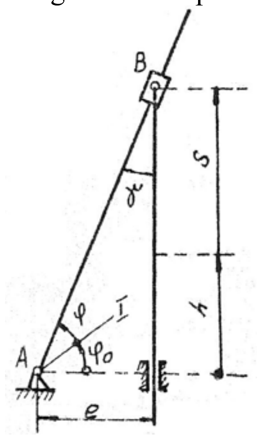

Fig. 2 Two crank guideways mechanism and its geometrical parameters

This mechanism contains two crank guideways: a rotating one having the AB direction and a sliding one having the CD direction. This mechanism is called tangential mechanism.

The mechanism has three geometrical parameters: *e, h* and φ_0.

Using the notations in figure 2, we can write:

$$h + s - e\,tg(\varphi_0 + \varphi) = 0 \qquad (30)$$

In order to determine the three parameters, the equation (30) can be written under the following polynomial:

$$p_0 + f_0(\varphi) + p_1 f_1(\varphi) + p_2 f_2(\varphi) = 0 \quad (31)$$

where

$$\begin{cases} p_0 = e\,tg\varphi_0 - h \\ p_1 = e + h\,tg\varphi_0 \\ p_2 = tg\varphi_0 \end{cases} \qquad (32)$$

$$\begin{cases} f_0 = -s \\ f_1 = tg\varphi \\ f_2 = s\,tg\varphi \end{cases} \qquad (33)$$

For a set of three values of angle φ ($\varphi_1,\varphi_2,\varphi_3$) in the approximation interval (φ_0,φ_m), considered as abscissas of interpolation nodes, arbitrarily chosen or by Chebyshev spacing, we can write three linear equations, from which we determine the coefficients p_j, $j=0,1,2$ using the relations (32). We determine the geometrical parameters, first $\varphi_0 = arctg p_2$, then *e* and *h* from the following system:

$$\begin{cases} e\,tg\varphi_0 - h = p_0 \\ e + h\,tg\varphi_0 = p_1 \end{cases} \qquad (34)$$

Error analysis

The relation (30) expresses the deviation between what the guideway can do and what has been imposed to. This deviation must verify the following relation, where Δs_{ad} represents the permissible deviation:

$$\Delta s(\varphi) = e\,tg(\varphi_0 + \varphi) - s - h \le \Delta s_{ad} \qquad (35)$$

Supplementary conditions

The transmission angle γ results directly from figure 2:

$$ctg\,\gamma = \frac{h+s}{e} \qquad (36)$$

In order to satisfy the condition of the minimum transmission angle, the following relation must be fulfilled:

$$h + s_{\max} \le e\,ctg\gamma_{ad} \qquad (37)$$

where s_{max} is the maximum displacement of the driven element, γ_{ad} is the permissible transmission angle, *h* is the distance between the reference axis and the initial position of slide B.

Earlier in the paper, the driving element is the rotating slotted link. In this case, the mechanism generates the function $f(\varphi;\ e,\ \varphi_0,\ h)$. When the sliding slotted link is the driving element, the mechanism can generate the identical function $f(s;\ e,\ \varphi_0,\ h)$. As a conclusion, the two crank guideways mechanism doesn't depend on the driving guideway.

Numerical examples

Let synthesize a slider-crank inverted linkage for four simple accuracy points to approximate the function

$$\psi(\varphi) = \varphi^2 / 25 \qquad (38)$$

on the interval $\varphi_0 = 0, \varphi_m = 50^\circ$.

Solution. The interpolation nodes are chosen by Chebyshev spacing:

$$\begin{cases} \varphi_1 = \frac{\varphi_m}{2}\left(1 - \cos\frac{\pi}{8}\right) = 1{,}903012^0 \\ \varphi_2 = \frac{\varphi_m}{2}\left(1 - \cos\frac{3\pi}{8}\right) = 15{,}432914^0 \\ \varphi_3 = \frac{\varphi_m}{2}\left(1 + \cos\frac{3\pi}{8}\right) = 34{,}567086^0 \\ \varphi_4 = \frac{\varphi_m}{2}\left(1 + \cos\frac{\pi}{8}\right) = 48{,}096988^0 \end{cases} \qquad (39)$$

The system (16) (with the explanatory relations (17) and (18)) of four unknowns *pj, j=0,1,2,3* is solved, obtaining the solutions:

$$\begin{cases} p_0 = 0.74368 \\ p_1 = 0.44078 \\ p_2 = -1.73155 \\ p_3 = -0.22851 \end{cases} \qquad (40)$$

The unknown geometrical parameters of the synthesized linkage are calculated in relations (19):

$$\begin{cases} \psi_0 = -12.87208^0 \\ e = 0.72499 \\ \varphi_0 = -27.15408^0 \\ a = -1.74187 \end{cases} \qquad (41)$$

We calculate ψ_r with relations (10)-(13) and with relation (15) we calculated the deviation $\Delta\psi(\varphi)$, which is compared to the allowable value for several values on the approximation interval (φ_0,φ_m), relation (15).

In order to render evident the way angle ψ_r (13) varies in relation to φ we shall present this function graphically together with the function $\psi(\varphi) = \varphi^2/25$. In figure 3 the angle ψ_r is shown with the red colour and the angle $\psi(\varphi) = \varphi^2/25$ in the interval (φ_0,φ_m) is shown with the blue colour.

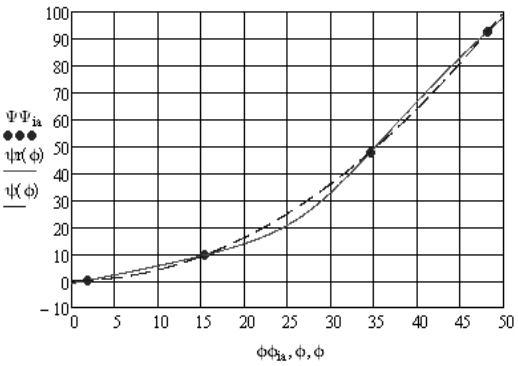

Fig. 3 The variation of the angles ψ_r and ψ in relation to φ

The way deviation $\Delta\psi(\varphi)$ varies in relation to φ is shown graphically in the figure 4:

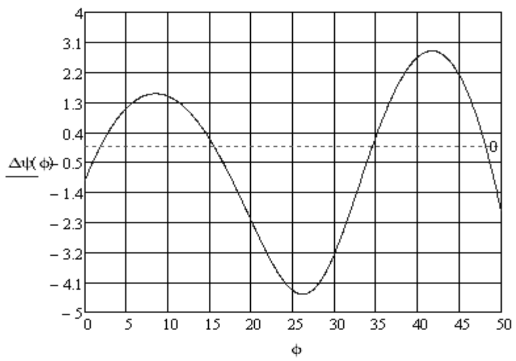

Fig. 4 The deviation $\Delta\psi(\varphi)$ in relation to φ

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