THE POSITIONAL SYNTHESIS OF THE SLIDER-CRANK MECHANISM

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Abstract. In the paper we present the positional synthesis of the slider-crank linkage as part of mining machines and equipment.

Introduction

The notations in fig. 1 will be used for the slider-crank mechanism:

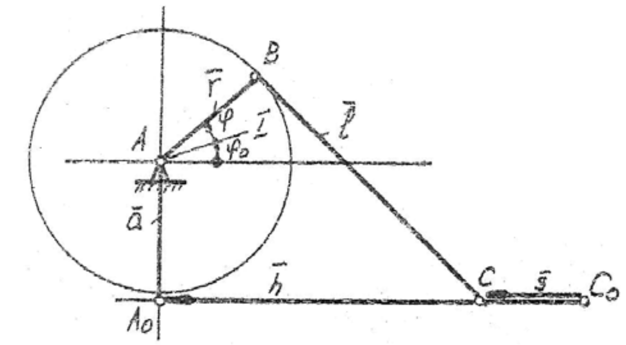

Fig. 1 The slider-crank mechanism and the parameters defining it

r – crank AB length;
l – connecting rod BC length;
$a = \overline{AA_0}$ - eccentricity value;
$s = \overline{C_0C}$ - slider displacement against original position C_0 (variable dimension);
$h = \overline{A_0C_0}$ - parameter indicating the initial position C_0 of the slider A_0 (constant dimension);
φ_0 – crank initial positioning angle, considered counterclockwise (constant dimension);
φ – crank rotation angle, measured in the same direction with φ_0, against the line AI (variable dimension).

The function to be approximated is the following: $s = F(\varphi)$.

The mechanism position equation is obtained writing the contour vectorial equation:

$$\bar{l} = \bar{a} + \bar{h} + \bar{r} - \bar{s} \qquad (1)$$

Squaring, we get:

$$s^2 + h^2 - 2sh - 2rh\cos(\varphi_0 + \varphi) + 2rs\cos(\varphi_0 + \varphi) + \\ + 2ar\sin(\varphi_0 + \varphi) + r^2 + a^2 - l^2 = 0 \qquad (2)$$

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The slider-crank mechanism is determined by five geometrical parameters *a, r, l, h* and φ_0. The same calculus model will be used for determining them as the one used for the synthesis of the four-bar linkage.

The calculation of three parameters

Equation (2) will be written as the following polynomial (similar to that used at the synthesis of the four-bar linkage):

$$p_0 + f_0(\varphi) + p_1 f_1(\varphi) + p_2 f_2(\varphi) = 0 \qquad (3)$$

The case of determining parameters a, h and l (r and φ_0 being chosen arbitrarily)

In order to facilitate calculations, it will be taken r = 1, considering *a, h* and *l* as relative values, obtained in relation to the crank length *r*. In this case the polynomial terms (3) have the following expressions:

$$\begin{cases} p_0 = \dfrac{a^2 + h^2 + 1 - l^2}{2} \\ p_1 = -h \\ p_2 = a \end{cases} \qquad (4)$$

$$\begin{cases} f_0(\varphi) = \dfrac{s^2}{2} + s\cos(\varphi_0 + \varphi) \\ f_1(\varphi) = s + s\cos(\varphi_0 + \varphi) \\ f_2(\varphi) = \sin(\varphi_0 + \varphi) \end{cases} \qquad (5)$$

The remaining calculations are similar to those in the case of the four-bar linkage.

The case of determining parameters a, r and l (h and φ_0 being chosen arbitrarily)

Parameter *h* will be considered equal to unit (*h=1*), and *a, r, l* taken as relative values in relation to *h*.

In this case, the terms of polynomial (3) have the following expressions:

$$\begin{cases} p_0 = r^2 + a^2 + 1 - l^2 \\ p_1 = 2r \\ p_2 = 2ar \end{cases} \qquad (6)$$

$$\begin{cases} f_0(\varphi) = s^2 - 2s \\ f_1(\varphi) = (1 - s)\cos(\varphi_0 + \varphi) \\ f_2(\varphi) = \sin(\varphi_0 + \varphi) \end{cases} \qquad (7)$$

Then we proceed as in the preceding case.

The calculus of four parameters

Determining parameters a, r, h and s (φ_0 being chosen arbitrarily)

Parameter φ_0 is chosen arbitrarily. In this case the interpolation polynomial is the following:

$$p_0 + f_0(\varphi) + p_1 f_1(\varphi) + p_2 f_2(\varphi) + \\ + p_3 f_3(\varphi) + p_4 f_4(\varphi) = 0 \tag{8}$$

and the terms used in relation (8) have the appearance:

$$\begin{cases} p_0 = \dfrac{l^2 - r^2 - a^2 - h^2}{2r} \\ p_1 = -a \\ p_2 = h \\ p_3 = -\dfrac{l}{r} \\ p_4 = p_2 p_3 = -\dfrac{hl}{r} \end{cases} \tag{9}$$

$$\begin{cases} f_0(\varphi) = -s\cos(\varphi_0 + \varphi) \\ f_1(\varphi) = \sin(\varphi_0 + \varphi) \\ f_2(\varphi) = -\cos(\varphi_0 + \varphi) \\ f_3(\varphi) = \dfrac{s^2}{2} \\ f_4(\varphi) = -s \end{cases} \tag{10}$$

The coefficient p_4, being a combination between coefficients p_2 and p_3, the system (10) becomes non-linear in $p_j; j=0,1,2....$

The case of determining parameters a, r, l and φ_0 (h being chosen arbitrarily)

Parameter h is chosen arbitrarily. Functions $sin(\varphi_0+\varphi)$ and $cos(\varphi_0+\varphi)$ are developed in equation (2). After ordering and grouping, the terms are identified with those of the interpolation polynomial (8). The following expressions are obtained for the terms involved:

$$\begin{cases} p_0 = \dfrac{l^2 - r^2 - a^2}{2ar\cos\varphi_0} \\ p_1 = \dfrac{1}{2ar\cos\varphi_0} \\ p_2 = -tg\varphi_0 \\ p_3 = \dfrac{1}{a} \\ p_4 = -\dfrac{tg\varphi_0}{a} = p_2 p_3 \end{cases} \tag{11}$$

$$\begin{cases} f_0(\varphi) = \sin\varphi \\ f_1(\varphi) = (h-s)^2 \\ f_2(\varphi) = \cos\varphi \\ f_3(\varphi) = (h-s)\cos\varphi \\ f_4(\varphi) = (h-s)\sin\varphi \end{cases} \tag{12}$$

The coefficient p_4 being expressed as product of the coefficients p_2 and p_3, the system will be non-linear in $p_j; j=0,1,2....$

The calculus of five parameters

In this case, the position equation is similar to the equation in the case of the four-bar linkage:

$$p_0 + f_0(\varphi) + p_1 f_1(\varphi) + p_2 f_2(\varphi) + \\ + p_3 f_3(\varphi) + p_4 f_4(\varphi) + p_5 f_5(\varphi) = 0 \tag{13}$$

The terms have the expressions:

$$\begin{cases} f_0(\varphi) = -\sin\varphi \\ f_1(\varphi) = s^2 \\ f_2(\varphi) = s \\ f_3(\varphi) = s\cos\varphi \\ f_4(\varphi) = \cos\varphi \\ f_5(\varphi) = \sin\varphi \end{cases} \tag{14}$$

$$\begin{cases} p_0 = \dfrac{a^2 + r^2 + h^2 - l^2}{2r\sin\varphi_0} \\ p_1 = -\dfrac{1}{2r\sin\varphi_0} \\ p_2 = -\dfrac{h}{r\sin\varphi_0} \\ p_3 = ctg\,\varphi_0 \\ p_4 = a - h\,ctg\,\varphi_0 \\ p_5 = p_3 p_4 - \dfrac{p_2}{2p_1}(1 + p_3^2) \end{cases} \tag{15}$$

In this case coefficient p_5 is a combination of four coefficients p_1, p_2, p_3, p_4, and the functions $f_i(\varphi)$ have a simple structure.

Then we proceed as in the previous case.

Remark. The calculation sequence for each case develops as follows:

- The abscissas of the interpolation nodes φ_j are chosen arbitrarily on the interval (φ_0, φ_m) or by Chebyshev spacing;
- The mechanism position equations are written for these points, obtaining linear systems in the coefficients p_j, $j=0,1,2,...$, which are solved;
- After the calculation of the coefficients p_j, $j=0,1,2,...$, the geometrical parameters required in each case are found. For example, for the case of five parameters, finding them has the following sequence:

$$\begin{cases} \varphi_0 = arc\,ctg\,p_3 \\ r = \dfrac{1}{2p_1\sin\varphi_0} \\ h = -\dfrac{p_2}{p_1} \\ a = p_4 + hp_3 \\ l = \sqrt{a^2 + r^2 + h^2 - 2rp_0\sin\varphi_0} \end{cases} \tag{16}$$

- The magnitude of the structured error is calculated (as it has been calculated for the four-bar linkage). In order to do it, the real displacement of the slider from the initial position C_0 is first found, using the following relation (fig.1):

$$s_r = h - r\cos(\varphi_0 + \varphi) \pm \sqrt{l^2 - [a + r\sin(\varphi_0 + \varphi)]^2} \quad (17)$$

- The value of the function to approximate $s=F(\varphi_0+\varphi)$ is calculated;
- The condition that the approximation deviation should satisfy the relation is checked:

$$\max|\Delta s| = |s - s_r| \le \Delta s_a \quad (18)$$

where Δs_a is the allowable deviation.

Supplementary conditions

As in the case of the four-bar linkage for the slider-crank, too, it is required that the relative length of the elements should be within two limits L and $\frac{1}{L}$, while the minimum transmission angle should not be less than the minimum allowable value.

Moreover, the crank condition is checked.

Let us consider the crank length equal to unit, $r=1$. The other relative lengths of the mechanism elements have to verify the following inequalities:

$$\begin{cases} a \le L; \quad a \ge \frac{l}{L} \\ a \ge \frac{1}{L} \quad l \le L \\ a \le L \cdot l; \quad l \ge \frac{1}{L} \end{cases} \quad (19)$$

The transmission angle of the mechanism γ is shown in figure 2. This is formed between the direction of point C absolute velocity V_C and the direction of relative velocity V_{CB}.

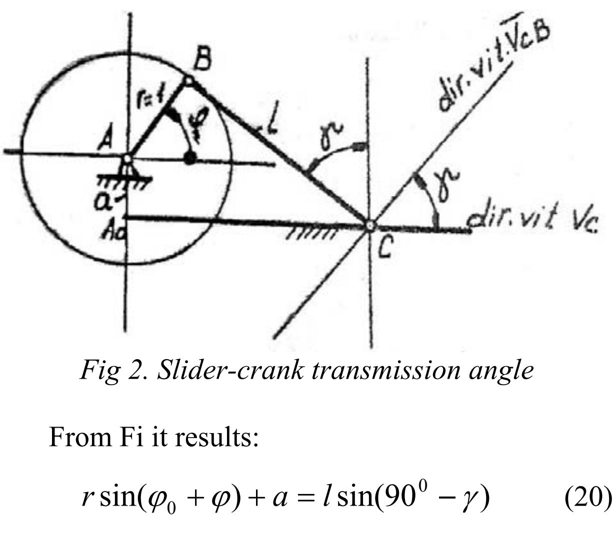

Fig 2. Slider-crank transmission angle

From Fi it results:

$$r\sin(\varphi_0 + \varphi) + a = l\sin(90^0 - \gamma) \quad (20)$$

$$\cos\gamma = \frac{a + \sin(\varphi_0 + \varphi)}{l} \quad (21)$$

From relation (21), knowing that $-1 \le \sin(\varphi_0 + \varphi) \le 1$, we deduce:

a) For a > 0:

$$\cos\gamma_{\min} = \frac{a+1}{l}; \cos\gamma_{\max} = \frac{a-1}{l} \quad (22)$$

b) For a < 0:

$$\cos\gamma_{\min} = -\frac{a+1}{l}; \cos\gamma_{\max} = \frac{1-a}{l} \quad (23)$$

Writing down the value of the allowable transmission angle with γ_{ad} and knowing that the cosinus function sign is of no importance, we infer:

$$l\cos\gamma_{ad} \le a + 1 \quad (24)$$

In order to have a crank, the alignment condition of the pairs B, A, C must be fulfilled (see fig. 3):

$$a < l - 1 \quad (25)$$

If condition (25) is not fulfilled, the mechanism will be of the slider-lever type.

From the inequalities (24) and (25) it results that for the slider-crank mechanism, the following inequalities are to be fulfilled so as to have a crank and available transmission angle:

$$l\cos\gamma_{ad} \le a + 1 < l \quad (26)$$

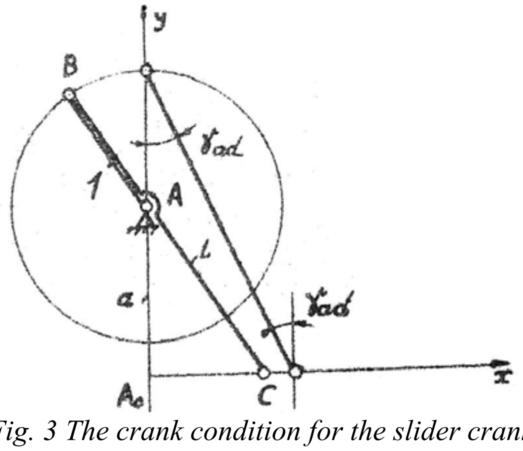

Fig. 3 The crank condition for the slider crank mechanism

Numerical example

Let us synthesize a slider crank mechanism for 5 accuracy points, for the function:

$$F(\varphi) = 1.145916 \cdot arctg(0.853531 \cdot \varphi) \quad (27)$$

on the interval $\varphi_0=0^0$, $\varphi_m=80^0$.

Solution The abscissa of the interpolation nodes are chosen by Chebyshev spacing:

$$\begin{cases} \varphi_3 = \dfrac{\varphi_0 + \varphi_m}{2} = 40^0 \\ \varphi_1 = \varphi_3 - \dfrac{\varphi_m - \varphi_0}{2}\cos\dfrac{\pi}{10} = 1,958^0 \\ \varphi_2 = \varphi_3 - \dfrac{\varphi_m - \varphi_0}{2}\cos\dfrac{3\pi}{10} = 16,489^0 \\ \varphi_4 = \varphi_3 + \dfrac{\varphi_m - \varphi_0}{2}\cos\dfrac{3\pi}{10} = 63,511^0 \\ \varphi_5 = \varphi_3 + \dfrac{\varphi_m - \varphi_0}{2}\cos\dfrac{\pi}{10} = 78,042^0 \end{cases} \quad (28)$$

The system (13) (with the explanatory relations (14) and (15)) of five linear equations with five unknowns $p_j,\ j = \overline{0,4}$ is solved, obtaining the solutions:

$$\begin{cases} p_0 = 0.879 \\ p_1 = 0.572 \\ p_2 = -1.773 \\ p_3 = 1.194 \\ p_4 = 0.314 \end{cases} \quad (29)$$

The unknown geometrical parameters of the synthesize linkage are calculated with relations (16) in the following sequence:

$$\begin{cases} \varphi_0 = 72.548^0 \\ r = 0.916 \\ h = 1.549 \\ a = -0.392 \\ l = 1.362 \end{cases} \quad (30)$$

Taking *n* values, inclusively the values of the nodal abscissa of s_i on the interval [s_0,s_m], with relation (17) the value of displacement s_r can be determined and then it is found the value of deviation

$$\Delta s(s_i) = f(s_i; a, r, l, \varphi_0, h) - F(s_i),\ i = \overline{1,n} \quad (31)$$

whose magnitude is compared to the values of the allowable deviation Δs_a, relation (18). In fig 4.are represented the approximate and approximated functions (which are, practically identical).

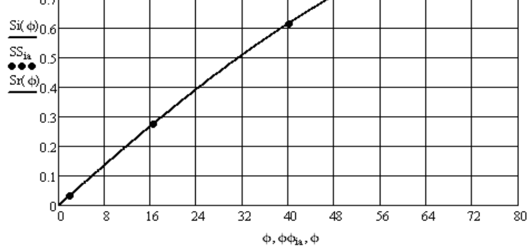

Fig. 4 The approximate and approximated functions

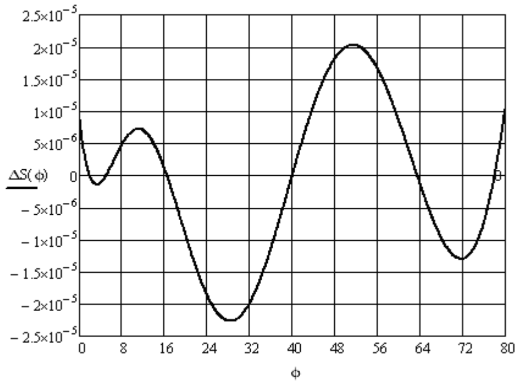

Fig. 5 The error curve

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