SYNTHESIS OF MECHANISMS BY THE INTERPOLATION METHOD

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The paper presents aspects of designing simple planar linkages as component parts of mining machines and equipment.

Keywords: mechanism synthetics, interpolation method, interpolation nodes

Introduction

The most common practical requirements for the design (synthesis) of a mechanism are:

- Creating a motion law demanded at a mechanism element, as a rule at one of the working elements;
- Plotting a given trajectory by one of the points of a mechanism's elements.

The accurate solving of the two types of problems is possible only with mechanisms with higher pairs (for example cam mechanisms), at which the synthesis is called exact.

With low pair mechanisms, due to the limited motions the low pairs allow (rotations and translations), the conditions imposed by the two synthesis aspects can be fulfilled only approximately and therefore their synthesis is called approximate.

Nevertheless, the low pair mechanism can create the law or the required trajectory accurately for a limited number of positions and with deviations for the remaining positions in the approximation interval.

On the synthesis of a mechanism, in order to create a given motion law at the working element it is required that depending on the working element motion law (1) defined by the relation $\varphi_1 = F_1(t)$ (in the case of the element with rotating motion) or by $S_1=G_1(t)$ (in the case of the element with translation motion), figure 1, we should design a mechanism at which the working element *n* is to have, on a certain working interval $[\varphi_{10}, \varphi_{1n}]$, respectively $[S_{10}, S_{1n}]$ a required motion, defined by the functions $\varphi_n = F_n(t)$ or $S_n=G_n(t)$.

Removing parameter *t* from the functions $\varphi_1(t)$, $S_1(t)$ and $\varphi_n(t)$, $s_n(t)$, the positional relations, shown in Table 1 are obtained.

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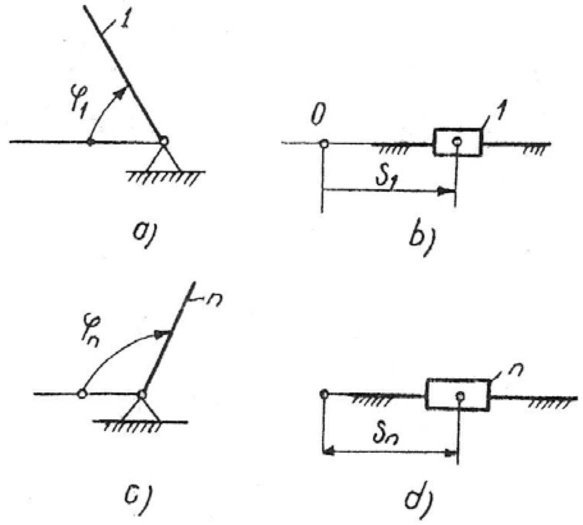

Figure 1 *Defining the motion of the input element and that of the output element:*
a) the input element with rotating motion;
b) the input element with translation motion;
c) the output element with rotating motion;
d) the output element with translation motion

Table 1

Input element motion	Output element motion	
	Rotation	Translation
Rotation $\varphi_1=F_1(t)$	$\varphi_n=F(\varphi_1)$	$S_n=F(\varphi_1)$
Translation $S_1=G(t)$	$\varphi_n=G(S_1)$	$S_n=G(S_1)$

The time function defining the motion of the working output element (n) in relation to the input element (1) can be rendered graphically or analytically.

Let us consider the case of correlating the rotations of two cranks in order to illustrate it figure 2.

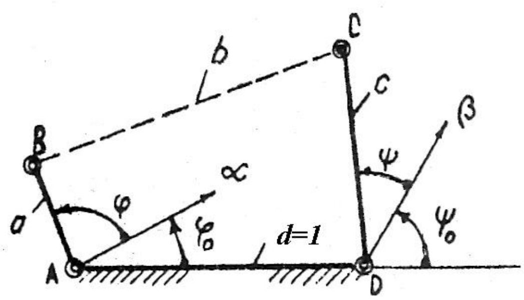

Figure 2 *Designing a mechanism according to the input and output elements positions*

The driving input element AB has its angular coordinate φ in relation to the arbitrary reference axis A-α, while the working output element CD has the angular coordinate ψ, considered in relation to the arbitrary reference axis D-β. The two reference axes A-α and D-β are oriented to the direction of the fixed element AD by the angle φ_0, respectively ψ_0.

The connection between the driving element AB and the working element CD can be made either by the connecting rod BC or, in a general case, by a kinematic chain made up of a certain number of links.

The mechanism will have *n* elements in all, which also include elements AB and CD.

The positional synthesis conditions require that the diagram of the mechanism positional function should pass through a series of points $Q_1(\varphi_1,\psi_1)$, $Q_2(\varphi_2,\psi_2)$, ... , figure 3. In some cases, in the points Q_1, Q_2, ..., the function shape is also defined by the transmission ratio $i = \frac{d\psi}{d\varphi}$ and its derivative $\frac{di}{d\varphi} = \frac{d^2\psi}{d\varphi^2}$.

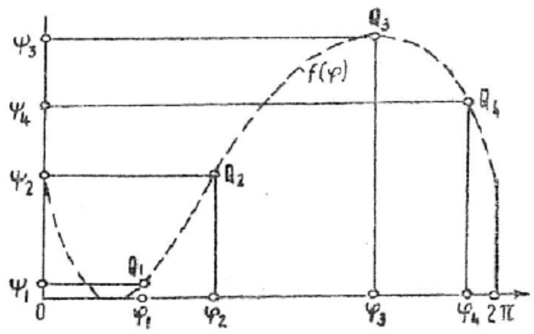

Figure 3 *The diagram of the position function*

Each point Q_j, *j=1,2,...,n* represents associated positions of the driving element and of the working element.

The problem can be solved either graphically or analytically. With the analytical solution, the number of equations that can be written should be equal to the number of unknowns.

For *n* mechanism positions can be written *2n* scalar contour equations, obtained by projecting the mechanism vectorial contour in its *n* positions onto the two axes of the reference system the mechanism is related to. Thus, for the four-bar linkage it is necessary to have 5 parameters: the relative lengths a, *b*, *c* and the coordinates *α* and *β* which position the reference axes of the driving element AB, respectively the working one CD, that is

$$2n = n + 5 \tag{1}$$

It results that in the case of the four-bar linkage no more than 5 positions relatively associated to the cranks (levers) AB and CD can be given.

Therefore, in the positional synthesis it is required to find that mechanism structure which, for a given structure of the working element

$$\psi = F(\varphi) \tag{2}$$

continues on the given interval $[\varphi_0, \varphi_m]$, figure 4, to be approximated as faithfully as possible.

This function will be rendered only approximately (in a limited number of positions) by the designed low pair mechanism.

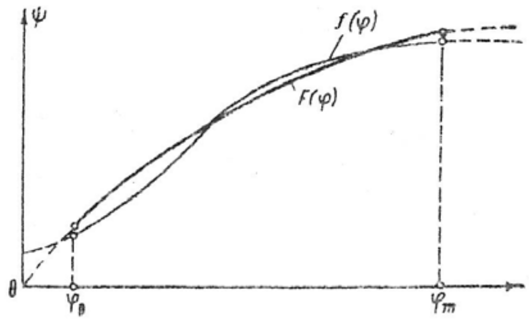

Figure 4 *Approximating a given function F(φ) by the function f(φ)*

The function the mechanism can create for certain chosen or calculated parameters (called position functions) has the following appearance in the case of the four-bar linkage:

$$\psi = f(\varphi; a, b, c, \varphi_0, \psi_0) \tag{3}$$

Approximation error

The function given by relation (3) has to be as close as possible to the one given by relation (2) for the four-bar linkage. The degree of closeness between the two functions can be appreciated by the difference Δψ:

$$\Delta\psi = f(\varphi) - F(\varphi) \tag{4}$$

called deviation (error) of the position function f(φ) from the function F(φ), Figure 5.

This deviation can be considered on any direction. In most cases, for simplification, it is considered on the directions of the coordinating axes.

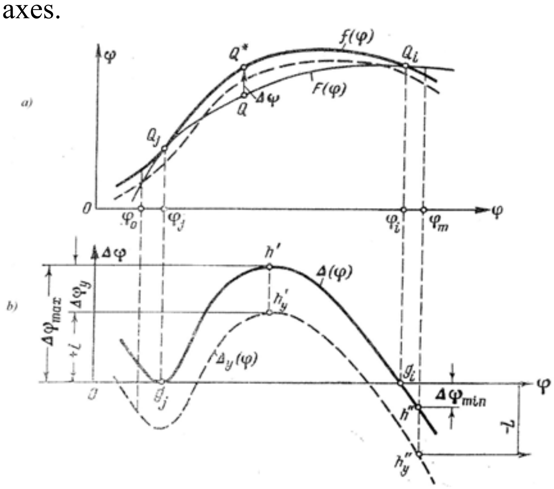

Figure 5 *The diagrams of the functions f(φ) and F(φ) and the diagrams of errors*

Thus, the vectors $\overline{\Delta\psi} = \overline{QQ^*}$, equal in magnitude to the distance between the diagram of the approximating function f(φ) and that of the approximated function F(φ) on the direction of the axis ψ and oriented from the diagram of the given function F(φ) towards the diagram of the mechanism position function f(φ) (figure 5 a), give the deviation (error) of the position function f(φ) from the given function F(φ) in any interval point.

The equation (3) shows the deviation Δψ is dependent on the coordinate φ of the driving element, that is:

$$\Delta\psi = \Delta(\varphi) \tag{5}$$

The diagram of the deviations Δψ depending on the position angle of the driving element is rendered in figure 5.b.

From figure 5 we deduce the following:

a) The diagrams f(φ) and F(φ) have two categories of contact points: intersection points (contact of 0 order) and multiple contact points (of higher order than zero);
b) The intersection point g_i of the deviation diagram Δ(φ) with abscissae axis corresponds to the intersection point Q_i of the diagrams f(φ) and F(φ);
c) The tangent point g_j of the diagram Δ(φ) with abscissae axis corresponds to the tangent point (contact point of order 1) Q_j of the two diagrams.

Writing down with $\Delta\psi_{max}$ and $\Delta\psi_{min}$ the maximum deviation, respectively the minimum deviation reached by the deviation diagram in h′ and h′′ on the interval $[\varphi_0,\varphi_m]$ and making their mean value, we get:

$$\Delta\psi_y = \frac{\Delta\psi_{max} + \Delta\psi_{min}}{2} \tag{6}$$

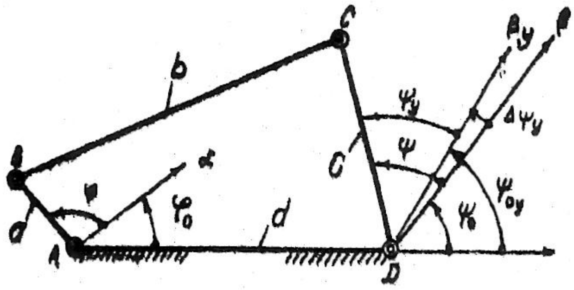

Figure 6 *Rotating the axes with a view to equalizing deviations*

If the axis D-β (see figure 6) is rotated round the point D with the angle $\Delta\psi_y$ (counterclockwise) it can be seen that the new coordinate of the axis D-β is:

$$\psi_y = \psi_0 + \Delta\psi_y \tag{7}$$

and the coordinate of the working element CD against the new axis D-β_y is:

$$\psi_y = \psi - \Delta\psi_y \tag{8}$$

Consequently, the diagram of the position function f(φ) being dependent on the parameters of the mechanism kinematic structure, will move on the ordonate axis direction on the distance ($-\Delta\psi_y$) to the position f(φ)y (see Figure 5b, dotted line). The curve of the given function does no depend on the mechanism parameters and it remains in the same position. Therefore, the error curve will have a displacement of the same magnitude ($-\Delta\psi_y$) on the ordonate direction to the position $\Delta_y(\varphi)$ plotted by a dotted line in Figure 5b.

The maximum and minimum points of the error curve h′ and h′′ will move to positions h_y' and h_y'' and will show new maximum and minimum values of the errors:

$$(\Delta\psi_y)_{max} = \Delta\psi_{max} - \Delta\psi_y = +\frac{\Delta\psi_{max} - \Delta\psi_{min}}{2} \tag{9}$$

$$(\Delta\psi_y)_{min} = \Delta\psi_{min} - \Delta\psi_y = -\frac{\Delta\psi_{max} - \Delta\psi_{min}}{2} \tag{10}$$

of equal magnitude:

$$L = \left| \frac{\Delta\psi_{max} - \Delta\psi_{min}}{2} \right| \tag{11}$$

The displacement $\Delta\psi_y$ is called equalizing error and its diagram $\Delta_y(\varphi)$ is called equalized error curve.

The magnitude value of the equalized errors (L) is used for estimating the accuracy of approximating the mechanism position function f(φ) against the given function F(φ). All the synthesis problems in which the magnitudes of the maximum ($\Delta\psi_{max}$) and minimum ($\Delta\psi_{min}$) errors are not equal between them are the cases most commonly met in practice.

Besides the errors of the linkage kinematic structure mentioned above, the machining and setting errors as well the clearance in the kinematic pairs, the rigidity of the elements etc. have obvious influence. In this paper we shall not deal with this kind of errors.

The problem of choosing the interpolation nodes

In the mechanism synthesis problems we use various analytical methods of approximating the functions [1,2,3,5,6,10,11]. The interpolation method is widely applied among these.

Within the calculus method by interpolation, some problems arise referring to the interpolation nodes:

- how many interpolation nodes have to be chosen?
- what kind of nodes, simple or multiple?
- where are they to be chosen in the imposed approximating interval so that the approximation error should be minimum?

Interpolation with simple nodes

Let us approximate the continuous function ψ=F(φ) on the interval [φ_0,φ_m] with a four-bar linkage. For example, for a four-bar mechanism, the problem lies in finding such values of the parameters a, b, c, φ_0 and ψ_0 and of the kinematic structure for which the position function $\psi = f(\varphi; a, b, c, \varphi_0, \psi_0)$ of the linkage on a given interval [φ_0,φ_m] should deviate as little as possible from the imposed function ψ=F(φ).

Practically, for the case of determining five parameters, we shall choose five points Q_1, …, Q_5, of abscissae φ_1, …, φ_5 on the approximating interval [φ_0,φ_m], where the two functions intersect. These are called simple interpolation nodes.

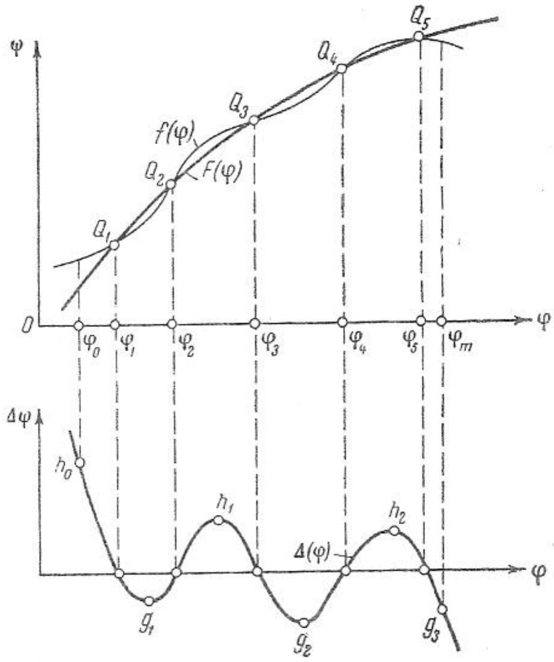

Figure 7 *Interpolation with simple nodes P-P-P-P-P*

The diagrams of the two functions and the plot of the approximation error Δψ are shown in figure 7. As the two plots F(φ) and f(φ) intersect, the error plot appears both on the positive side and the negative side of the axis ψ. Therefore the errors, in the case of interpolation with simple nodes, have different signs between the nodes on the approximation interval.

Interpolation with multiple nodes

Besides simple nodes, the interpolation method also uses multiple nodes, in which the plots f(φ) and F(φ) have common points of a certain established multiplicity order. When multiple nodes, which are equivalent to 2, 3, … simple nodes, are used, the number of precision points (nodes) on the approximating interval [φ_0,φ_m] diminishes from five with the multiplicity sum of the multiplicity nodes.

For example, in the case of the four-bar mechanism synthesis, for all the five parameters of the kinematic structure, two double nodes and a simple one can be chosen (in which case the nodes number on the approximating interval falls from 5 to 3 – 2 being the multiplicity sum of the two double nodes – as a double node has multiplicity one, figure 8) or other combinations of multiple or simple nodes.

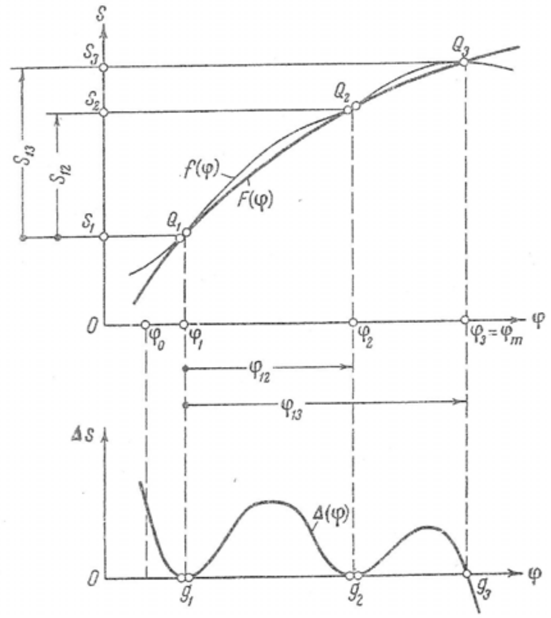

Figure 8 *Interpolation with two double and a simple one (PP-PP-P)*

The possible combinations between multiple and simple nodes for determining all the 5 geometric parameters of the four-bar mechanism structure are:

P-P-P-P-P; PP-P-P-P; P-PP-P-P;
PP-PP-P; PP-P-PP; PPP-P-P;
P-PPP-P; PPP-PP; PPPP-P; PPPPP

The symbols used [7,8,9] have the following significance:

PP – double node or of multiplicity one, in which two conditions k=2 are fulfilled: the value equality of the functions and the admission of the same tangent;

PPP – triple node or of the multiplicity two, in which three conditions are fulfilled k=3: value equality, the admission of the same tangent and of the same curvature;

PPPP – quadruple node or of multiplicity three, in which four conditions are fulfilled k=4: value equality of functions and the admission of the same tangent, curvature and supercurvature 1;

PPPPP – quadruple node or of the multiplicity three, in which all the conditions of the preceding case are fulfilled and besides the admission of supercurvature 2, k=5.

The above symbols have the following expression in the error function (4):

$$\begin{cases} \Delta\psi(\varphi) = \Delta'(\varphi) = 0; \\ \Delta\psi(\varphi) = \Delta'(\varphi) = \Delta''\psi(\varphi) = 0 \\ \vdots \\ \Delta\psi(\varphi) = \Delta'(\varphi) = \Delta''\psi(\varphi) = \ldots = \Delta^{(k-1)}(\varphi) = 0 \end{cases} \tag{12}$$

From Figure 8 it results that in the double nodes Q_1 and Q_2 the diagrams of the two functions are tangent. This is shown by two points infinitely close (one point for one condition) marked in each of the points Q_1 and Q_2. The simple node Q_3, the condition marking an intersection of two diagrams, is represented by a single point.

In the error diagram, there are the points g_1 and g_2 corresponding to the nodes Q_1 and Q_2. In them the error diagram is tangent to the abscissae axis; they are shown in the same way, by the two infinitely close points.

As we have only double nodes inside the interval, the errors on the wholes interval are of the same sign.

What is the difference between the interpolation with double nodes and the interpolation with simple nodes?

From the viewpoint of approximation accuracy, which we have agreed to estimate by mean error, the two situations are equivalent.

From the viewpoint of the amount of calculus (determining the five parameters of the four-bar linkage) the case of using double nodes is more advantageous.

The use of double nodes, for example, reduces the synthesis problem of the four-bar linkage for five relatively associated positions to three relatively associated positions.

This can be easily shown in the following.

Let us approximate the function $\psi=F(\varphi)$ with the help of the four-bar linkage on the interval $[\varphi_0,\varphi_m]$, using the interpolation with two double nodes and a simple one.

Choosing the abscissae of the nodes Q_1 and Q_2 so that $0 < \varphi_1 < \varphi_2 < \varphi_m$ and the abscissa of the simple node $\varphi_3=\varphi_m$, the ordinates of the given function are calculated:

$$\psi_i = F(\varphi_i), \quad i = 1,2,3 \tag{13}$$

The difference between the abscissae and the ordonates of the nodes Q_2 and Q_1, respectively Q_3 and Q_1 is calculated:

$$\varphi_{12} = \varphi_2 - \varphi_1; \quad \psi_{12} = F(\varphi_2) - F(\varphi_1) \tag{14}$$

$$\varphi_{13} = \varphi_3 - \varphi_1; \quad \psi_{13} = F(\varphi_3) - F(\varphi_1) \tag{15}$$

Let us suppose that we have found the parameters a, b, c, φ_0 and ψ_0 of the kinematic structure of the four-bar linkage for which the position function fulfills the conditions shown in Figure 9.

Let be the mechanism thus found in the three positions AB_1C_1D, AB_2C_2D, AB_3C_3D, corresponding to the three abscissa nodes Q_1, Q_2, Q_3: φ_1, φ_2, φ_3, see figure 9.

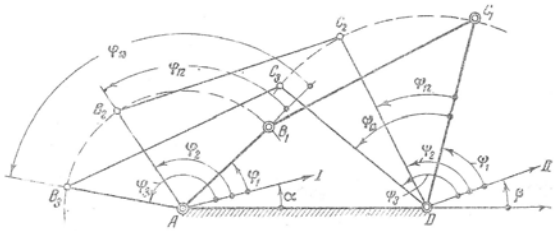

Figure 9 *The results of the four-bar linkage positional synthesis by the interpolation method with two double nodes and a simple one*

As the diagrams of two functions in the nodes Q_1 and Q_2 have order 1 contact, it means that the transmission ratio i_1 and i_2 in the points of abscissa φ_1 and φ_2 are also to be considered:

$$i_1 = F'(\varphi_1); \quad i_2 = F'(\varphi_2) \tag{16}$$

It is evident that the four-bar linkage synthesis, under the conditions mentioned, can be accomplished calculating the angles of the relative position of the driving and working elements with relations (14) and (15) and the transmission ratios corresponding to the first and the second positions with relation (16).

Interpolation with double nodes

At the arbitrary choice of abscissae φ_1 and φ_2 for the double nodes Q_1 and Q_2, the maximum errors $\Delta\psi_1$ and $\Delta\psi_2$ in the points h_1 and h_2 are not obtained equal as a rule.

If the abscissa φ_3 of node Q_3 is maintained unchanged abscissae φ_1 and φ_2 are modified so that the errors in the maximum points become equal:

$$\Delta\psi_0 = \Delta\psi_1 = \Delta\psi_2 \tag{17}$$

These are the least ultimate deviations that can appear on the given interval $[\varphi_0,\varphi_m]$ for any other abscissae of the nodal points Q1 and Q_2.

In figure 10 is shown the case where the conditions (18) are fulfilled.

By equalizing the diagram represented by broken line is obtained. It has six ultimate deviations, equal among them, in absolute value, but alternative as sign on the mentioned interval.

The greatest magnitude of these equalized deviations (L) is less than the greatest mean magnitude of the approximation deviations with other abscissae φ_1 and φ_2 for the double nodes Q_1 and Q_2.

In the Chebyshev polynomial theory we find very useful suggestions for choosing the abscissae φ_1 and φ_2 for which deviations $\Delta\psi_0$, $\Delta\psi_1$ and $\Delta\psi_2$ differ the least among themselves on the interval $[\varphi_0,\varphi_m]$.

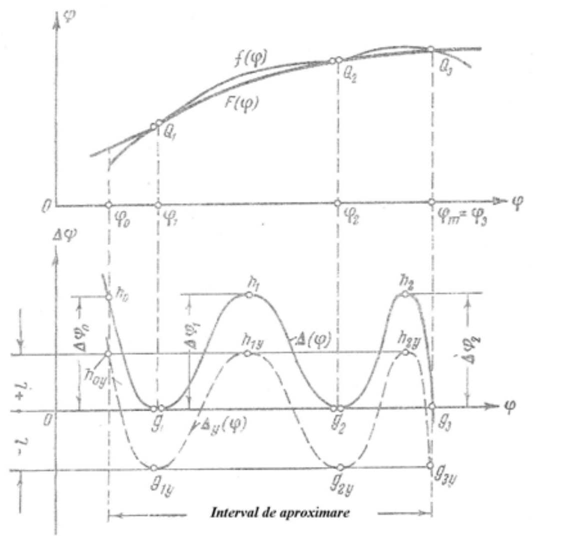

Figure 10 *Equalizing the deviations at the interpolation with two double nodes and a simple one (PP-PP-P)*

Based on what Chebyshev has established, the deviation of the approximating function f(φ) in relation with the given function F(φ) on the interval $[\varphi_0,\varphi_m]$ can be expressed by a polynomial of *n* degree:

$$\Delta(\varphi) = f(\varphi) - F(\varphi) = \varphi^n + a_{n-1}\varphi^{n-1} + ... + a_1\varphi + a_0 \quad (18)$$

whose coefficients a_{n-1}, a_{n-2}, ..., a_1, a_0 are determined from the conditions of the uniform approximation. From figure 11 it can de deduced that the deviation diagram $\Delta\psi$ has (n+1) points on the interval $[\varphi_0,\varphi_m]$ where the deviations have boundary values. These points, like those intersecting the abscissae axis, have a strictly logical spacing along the abscissae axis.

The abscissae of these points can be found geometrically by the construction in figure 11, where the case n=5 is illustrated. For this case, Chebyshev polynomial has degree 5, that is:

$$\Delta(\varphi) = \varphi^5 + a_4\varphi^4 + ... + a_1\varphi + a_0 \quad (19)$$

For simplification the interval $[0,\varphi_m]$ and not the interval $[\varphi_0,\varphi_m]$ has been taken into account.

The interval $[0,\varphi_m]$ is considered the diameter of a circumference divided into n=5 equal parts through the points 0, 1, 2, 3, 4, 5.

The abscissae of the points h_0, h_1, h_2 where the deviations are maximum are the same with those of the points 0, 2, 4, while the points where the deviations are minimum have the same abscissae as the points 1, 3, 5. The points on the circumference that indicate the middle of the arcs obtained through the points 0, 1, 2, 3 ,4, 5 have the abscissae with the intersection points of the diagram with abscissae axis, that is with the zero points on the deviations $\Delta\psi$ diagram.

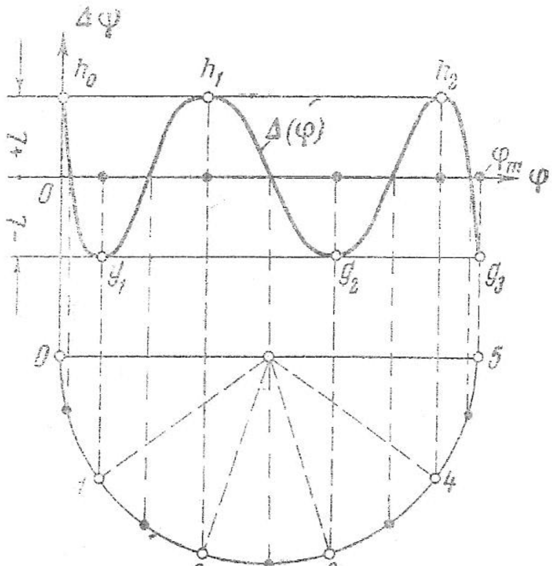

Figure 11 *The choice of interpolation nodes by Chebyshev spacing*

From figure 11 it can be noticed that in the case of uniform approximation, the points where the deviations take boundary values and the zero points of the deviations function alternate among themselves and towards the interval ends become more crowded.

Geometrically, from the construction shown in figure 11, the formulas are deduced:

$$\begin{cases} \varphi_1 = \dfrac{1}{2}\left(1 - \cos\dfrac{\pi}{5}\right)\varphi_m = 0{,}0955\varphi_m \\ \varphi_2 = \dfrac{1}{2}\left(1 - \cos\dfrac{3\pi}{5}\right)\varphi_m = 0{,}6545\varphi_m \\ \varphi_3 = \varphi_m \end{cases} \quad (20)$$

for the interval $[0,\varphi_m]$.

For the interval $[\varphi_0,\varphi_m]$, the formulae are:

$$\begin{cases} \varphi_1 = \varphi_0 + 0{,}0955(\varphi_m - \varphi_0) \\ \varphi_2 = \varphi_0 + 0{,}6545(\varphi_m - \varphi_0) \\ \varphi_3 = \varphi_m \end{cases} \quad (21)$$

Corresponding to them, the abscissae of the points h_1 and h_2 are given by the formulae:

$$\begin{cases} \varphi_1^{'} = \frac{1}{2}\left(1 - \cos\frac{2\pi}{5}\right)\varphi_m = 0{,}3455\varphi_m \\ \varphi_2^{'} = \frac{1}{2}\left(1 - \cos\frac{4\pi}{5}\right)\varphi_m = 0{,}9045\varphi_m \end{cases} \quad (22)$$

for the interval $[0,\varphi_m]$ and

$$\begin{cases} \varphi_1^{'} = \varphi_0 + 0{,}3455(\varphi_m - \varphi_0) \\ \varphi_2^{'} = \varphi_0 + 0{,}9045(\varphi_m - \varphi_0) \end{cases} \quad (23)$$

for the interval $[\varphi_0,\varphi_m]$.

Conclusions

The problem of designing a plane linkage with low pairs, which has to comply with the motion law of the output working element, defined by the function F (φ) expressed graphically, tabularly or analytically, lies in finding the n constant dimensional parameters contained in function f(φ), which have to be as close to possible to the given function F(φ).

If the number of parameters that determine the approximating function in the case of the four-bar linkage $\psi=f(\varphi;a,b,c,\varphi_0,\psi_0)$ is equal to the number of points chosen for the interpolation (n+1), then, generally speaking, the function f(φ) could be chosen so that the error relation (4) should become zero in the (n+1) points:

$$\Delta\varphi_i = f(\varphi_i) - F(\varphi_i); \quad i = 0,1,2,...,n \quad (24)$$

The function f(φ) can be written as a generalized polynomial of the type:]

$$f(\varphi_i) \equiv p_0 f_0(\varphi_i) + p_1 f_1(\varphi_i) + ... + p_n f_n(\varphi_i) \quad (25)$$

where p_0, p_1, …, p_n are explicit functions with m unknown mechanism parameters (the functions $f_j(\varphi_i)$ are known depending on the independent φ_i parameter).

The system (25) becomes a system of linear equations in p_j; j=0,1,…,n:

$$p_0 f_0(\varphi_i) + p_1 f_1(\varphi_i) + ... + p_n f_n(\varphi_i) = F(\varphi_i); \quad (26)$$
$$i = 0,1,...,n$$

From the system (26), in the case the number of (n+1) positions of the mechanism is equal to the number of dimensional parameters m, the parameters p_j (j=0,1,2,…,n) can be found, whereby the m parameters of the mechanism are obtained.

For the case when $r = n + 1 < m$, *m-r* parameters are chosen, that is *m-r* coefficients p_j can be chosen arbitrarily, the remaining *r* coefficients being determined from the system (26) of *r* equations with *r* unknowns.

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