MOTION ANALYSIS AND TECHNOLOGICAL VOLUME EVALUATION OF A HYDRAULIC EXCAVATOR

FLORIN DUMITRU POPESCU[1], ILDIKO BRÎNAŞ[2], CONSTATIN LAURENȚIU BRÎNAŞ[3]

Abstract: This paper presents the kinematic analysis and simulation of a hydraulically actuated one-bucket excavator modeled as a multi-degree-of-freedom manipulator. The main structural components of the excavator, consisting of the boom, stick, and bucket, are described, followed by an overview of three-dimensional manipulator kinematics based on homogeneous transformation matrices and the Denavit–Hartenberg formalism. The excavator working equipment is analyzed as an open kinematic chain with four degrees of freedom driven by hydraulic cylinders. Direct kinematics is used to determine the position and orientation of the bucket in the working space. A kinematic simulation of the excavation and unloading processes is performed using the SOLIDWORKS Motion Analysis module, where the hydraulic actuators are modeled as linear motors controlled by time-dependent STEP functions. The trajectory of a bucket tooth is evaluated, and the technological excavation volume is determined using two independent methods. The close agreement between the results validates the proposed modeling and simulation approach.

Keywords: Hydraulic excavators; Kinematic analysis; Denavit–Hartenberg method; Motion simulation; Technological excavation volume.

1. DESIGN FEATURES OF ONE-BUCKET EXCAVATORS

The component parts of a bucket excavator are presented in Figure 1. Here, the lower frame or lower platform can be distinguished, equipped with the travelling mechanism A, which supports all assemblies and mechanisms of the excavator and provides its movement. The rotating platform B is supported on the lower frame A by means of rollers or ball bearings. Mounted on this platform are the drive motor, the bucket lifting mechanisms, the excavator slewing mechanism, the thrust mechanism that

[1] *Prof. Ph.D. Eng., University of Petroşani, fpopescu@gmail.com*
[2] *Assoc. Prof. Ph.D. Eng, University of Petroşani*
[3] *Ph.D. Student Eng., University of Petroşani*

determines the variation in the length of the bucket arm during the loading and unloading processes, the actuation systems, and the operator's cab. The working equipment C consists of the boom 1, the arm 2, and the bucket 3, which, as previously mentioned, represents the working tool of the excavator.

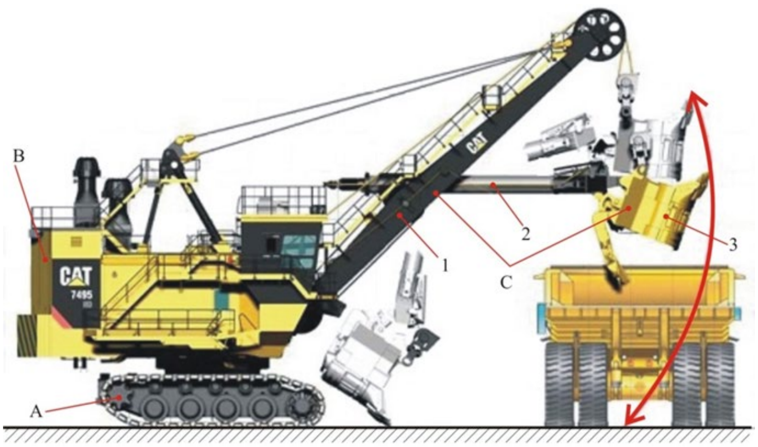

Fig. 1. Diagram of hydraulic excavator with direct bucket working equipment

2. KINEMATIC ANALYSIS OF 3D MANIPULATORS

The orientation and positioning in the working space of the execution element of a machine tool is achieved with the help of manipulators and industrial robots. The positioning of the working tool at any point in the three-dimensional space involves elementary independent translational or rotational motions, which are generated by the drive kinematic joints.

The positioning of the working tool is ensured by the trajectory-generating mechanism. This aspect highlights the fact that the trajectory-generating mechanism of the manipulator has at most three drive kinematic joints. The orientation of the working tool generally requires three independent rotational motions. This means that the orientation mechanism can also have at most three drive kinematic joints. From the above statements it follows that a manipulator, within an open kinematic chain, can have at most six drive kinematic joints.

The variable magnitudes of the drive kinematic joints (the angular displacements for rotary-type joints and the linear displacements for translation-type joins) defined the kinematic position. The kinematic positional analysis can be direct or inverse. Through direct positional kinematic analysis, the determination of the position and orientation of the working tool relative to the reference system attached to the manipulator base is desired, when the angular and translational displacements of the drive kinematic joints are known. The kinematic analysis is performed using homogeneous transformation matrices between a reference system attached to the manipulator base and a mobile reference system of the working tool.

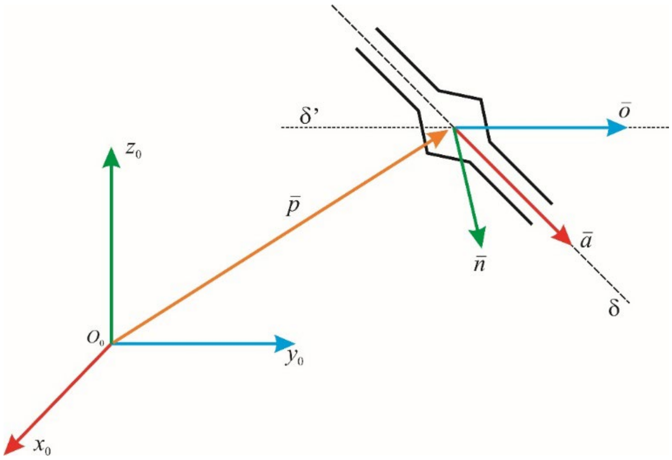

Fig. 2. Fixed reference system and mobile reference system

The mobile reference system is defined by three orthogonal unit vectors, and as can be seen in Figure 2:

$\overline{a} = \{a_x \quad a_y \quad a_z\}^T$ - approach vector, having the direction of the characteristic straight line (δ);

$\overline{o} = \{o_x \quad o_y \quad o_z\}^T$ - orientation vector, having the direction of the auxiliary straight line (δ');

$\overline{n} = \{n_x \quad n_y \quad n_z\}^T$ - vector that defines the orthogonal basis, $\overline{n} = \overline{o} \times \overline{a}$.

In accordance with the relation ${}^0T_j = {}^0T_i \cdot {}^iT_j$ the transition from the reference system attached to the working tool to the reference system attached to the manipulator base is carried out using the following transformation matrices:

$$ {}^0T_6 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \cdot {}^4T_5 \cdot {}^5T_6 = \prod_{i=1}^{6} {}^{i-1}T_i \tag{1} $$

The position and orientation of the working tool relative to the reference system attached to the manipulator base are expressed by the pose matrix defined as follows:

$$ R_{T_{EF}} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \overline{n} & \overline{o} & \overline{a} & \overline{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

where $\overline{p} = \{p_x \quad p_y \quad p_z\}^T$ is the position vector of the origin of the reference system attached to the working tool, and $\begin{bmatrix}\bar{n} & \bar{o} & \bar{a}\end{bmatrix}_{3\times 3}$ is an orthonormal orientation submatrix that has the following properties:

- the dot product of two-unit vectors is zero:

$$\bar{n}\cdot\bar{o}=\bar{o}\cdot\bar{a}=\bar{a}\cdot\bar{n}=0 \tag{3}$$

- the dot product of a unit vector with itself is unity:

$$\bar{n}\cdot\bar{n}=\bar{o}\cdot\bar{o}=\bar{a}\cdot\bar{a}=1 \tag{4}$$

- the cross product of two different unit vectors leads to the third one through cyclic permutations:

$$\bar{n}=\bar{o}\times\bar{a} \quad \bar{o}=\bar{a}\times\bar{n} \quad \bar{a}=\bar{n}\times\bar{o} \tag{5}$$

- the determinant of the orientation submatrix is unity:

$$\begin{vmatrix}\bar{n} & \bar{o} & \bar{a}\end{vmatrix}_{3\times 3}=1 \tag{6}$$

By equating the relation (2) with (1), the following is obtained:

$$\begin{bmatrix}\bar{n} & \bar{o} & \bar{a} & \bar{p}\\ 0 & 0 & 0 & 1\end{bmatrix}={}^{0}T_{6} \tag{7}$$

The development of the matrix product of the right-hand member of relation (7) requires the knowledge of the dimension of the elements for approaching the Denavit-Hartenberg formalism:

- for a driving rotary kinematic joint:

$$d_i, a_i, \alpha_i \ \textit{are constant and}\ \theta_i=\theta_i(t) \text{ is variable} \tag{8}$$

- for a driving translation kinematic joint:

$$\theta_i, a_i, \alpha_i \ \text{are constant and}\ d_i=d_i(t) \ \text{is variable} \tag{9}$$

Thus, each element f_{ij} of the matrix product, different from 0 or 1, will be a function that depends on the four parameters:

$$f_{ij}=f_{ij}(\theta_i, d_i, a_i, \alpha_i) \tag{10}$$

If relations (8) and (9) are considered, the following dependences on the generalized coordinate $q_i=q_i(t)$ are obtained, in the form:

$$f_{ij} = f_{ij}\left(q_1, q_2, \ldots, q_6\right) \quad i = 1, \ldots, 3; \quad j = 1, \ldots, 4 \tag{11}$$

The determination of the pose matrix assumes the solving of the direct positional kinematic problem, as a function of the generalized coordinates:

$$\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

3. POSITIONAL KINEMATICS OF THE HYDRAULIC EXCAVATOR

In Figure 3, the diagram of the working equipment of a hydraulically actuated excavator is presented. The working equipment of the hydraulic excavator operates similarly to a manipulator with four degrees of freedom.

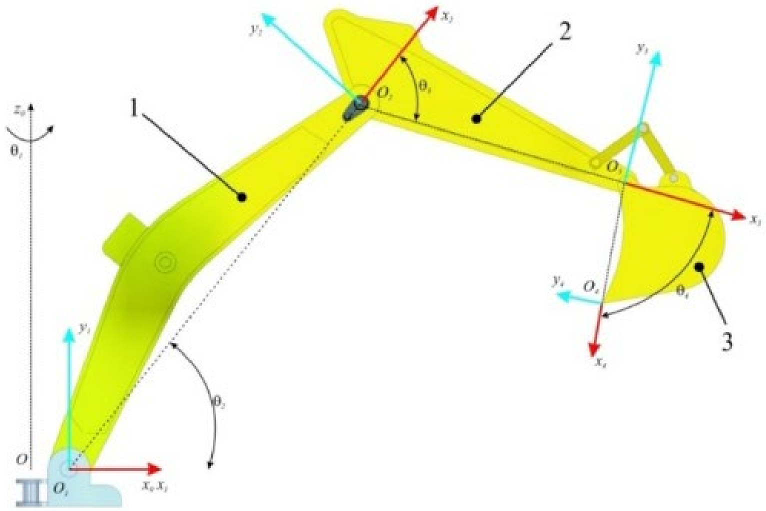

Fig. 3. Coordinate system attached to the excavator's elements

During motion, the excavator bucket moves along a trajectory described by its position in the working space, depending on the values of the rotation angles in the kinematic joints. The magnitudes of the rotation angles are determined by the magnitude of the displacements of the rods of the hydraulic actuation cylinders. These fulfill the role of the actuators. Thus, the hydraulic cylinders together with their control and command system represent the hydraulic actuators.

In order to establish the kinematic parameters of the excavator's elements in the sense of applying the Denavit-Hartenberg convention, in the first stage the reference system is necessary for describing the position of the bucket, that is, for describing the position of the working tool. For this purpose, a right-handed Cartesian coordinate reference system is placed on the excavator body, fixed $\text{Ref}(O_0, x_0, y_0, z_0)$ and in each kinematic joint a reference system $\text{Ref}(O_{i-1}, x_{i-1}, y_{i-1}, z_{i-1})$ is assigned.

An automatic excavation process must position and orient the bucket relative to the fixed reference system at any point within the working space. This is effectively achieved through the rotational motions in the kinematic joints, rotations that are obtained by modifying the lengths of the hydraulic actuators (Figure 4).

The direct kinematics analysis, having as input data the values of the rotation angles or the lengths of the hydraulic actuators, leads to the determination of the orientation and position of the bucket in the working space of excavator. Through inverse kinematics analysis, starting from the knowledge of the position and orientation of the bucket, the value of the rotation angles in the kinematics joints as well as the length of the hydraulics actuators can be determined.

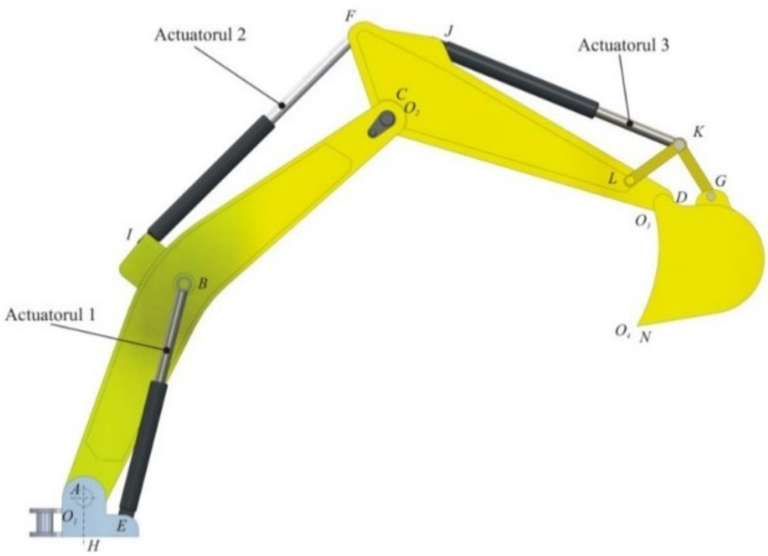

Fig. 4. Position of the hydraulic actuators

4. SIMULATION OF THE KINEMATICS OF A ONE-BUCKET EXCAVATOR

The study of the kinematics and dynamics of the mini excavator boom model during the excavation process was carried out in the SOLIDWORKS application using the **Motion Analysis** study type from the **Motion Study** menu. This menu provides the possibility for an assembly to assign rotary or linear actuators, for which the motion equations can be expressed as constant velocities, distance traveled over certain time intervals, periodic oscillations, trajectory segments, points, as well as by mathematical expressions.

In order to simulate both the excavation process and the material unloading process, each of the three hydraulic drive cylinders was assigned a linear motor that actuates between the piston rod end the base of the cylinder. The motion equations were introduced by mathematical expressions using the `STEP` function implemented in the SOLIDWORKS software library. In equations (13) analytical expression of this function is presented:

$$\text{STEP}\left(a, x_1, y_1, x_2, y_2\right) = \begin{cases} y_1, & a \le x_1 \\ y_1 + \left(x_2 - x_1\right)\left(3 - 2z\right)z^2, & x_1 \le a \le x_2 \\ y_2, & a \ge x_2 \end{cases} \qquad z = \frac{a - x_1}{x_2 - x_1} \quad (13)$$

The attachment of the linear drive motor of the hydraulic cylinder for lifting the boom is presented in Figure 5. As previously shown, the motion of this actuator is described by mathematical expressions. Thus, in Figure 6, the menu through which the mathematical expression of the motion modeling is implemented in the kinematic study is highlighted. For all three linear drive motors, the displacement of the piston rod relative to the base of the hydraulic cylinder as a function of time was described.

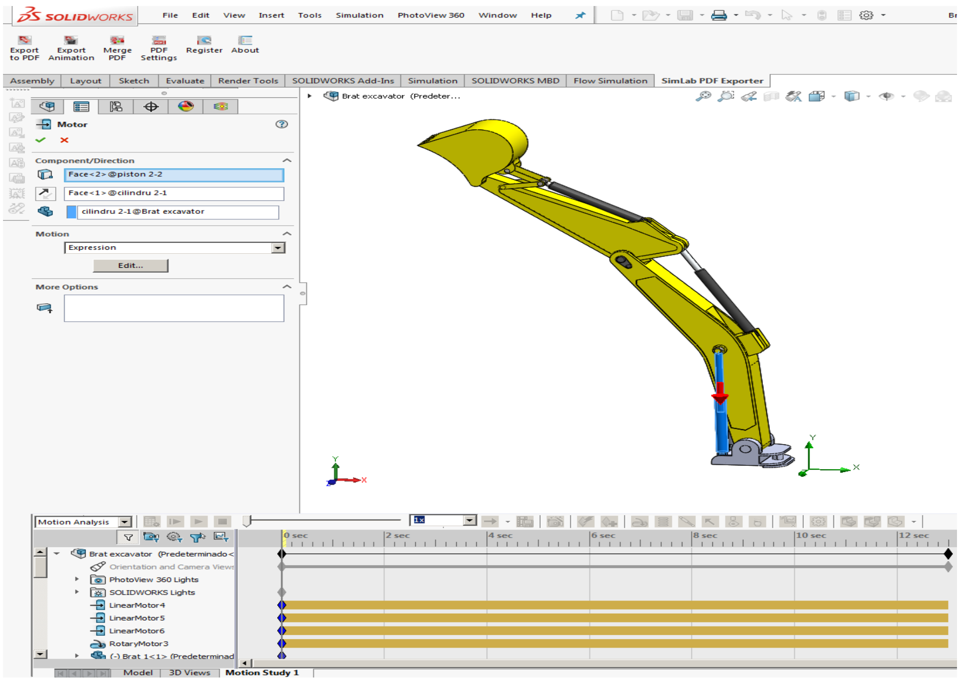

Fig. 5. Attachment of the linear drive motor of the hydraulic cylinder for lifting the boom

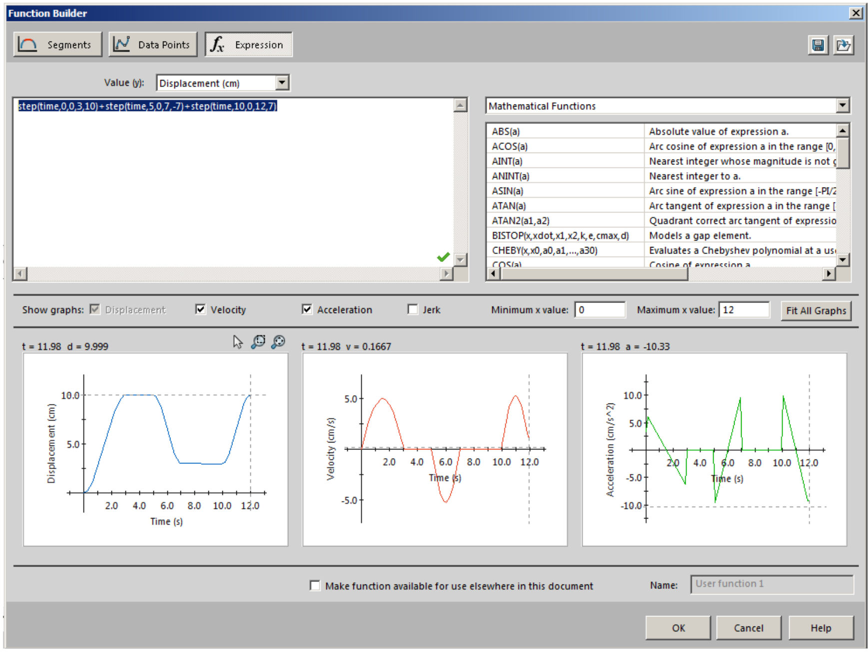

Fig. 6. Menu for implementing the mathematical expression describing the motion of the hydraulic cylinder for lifting the boom

After introducing relation (13), as can be seen in Figure 6, the variation graph of the displacement as a function of time is plotted, and by numerical derivation, the variation graphs of velocity and acceleration are also obtained. It should be emphasized that the `STEP` function ensures transitions of the piston displacement variation curve for the following considered time intervals: (0,3), (3,5), (5,7), (7,10) and (10,12) seconds.

$$\text{step}\left(time,0,0,3,10\right)+\text{step}\left(time,5,0,7,\text{-}7\right)+\text{step}\left(time,10,0,12,7\right) \quad (14)$$

In Figure 7, the attachment of the linear drive motor of the hydraulic cylinder that determines the motion of the arm itself to the arm assembly is presented, and in Figure 8 the menu through which the mathematical expression for modeling this motion is implemented in the kinematic study is highlighted. As in the case of the linear motor for lifting the boom, after introducing relation (14), the variation graph of the displacement as a function of time is plotted. By numerical derivation, the variation graphs of velocity and acceleration are also obtained. The `STEP` function ensures transitions of the piston displacement variation curve for the following considered time intervals: (1,5), (5,10), and (10,13) seconds.

$$\text{step}\left(time,1,0,5,55\right)+\text{step}\left(time,10,0,13,\text{-}43\right) \quad (15)$$

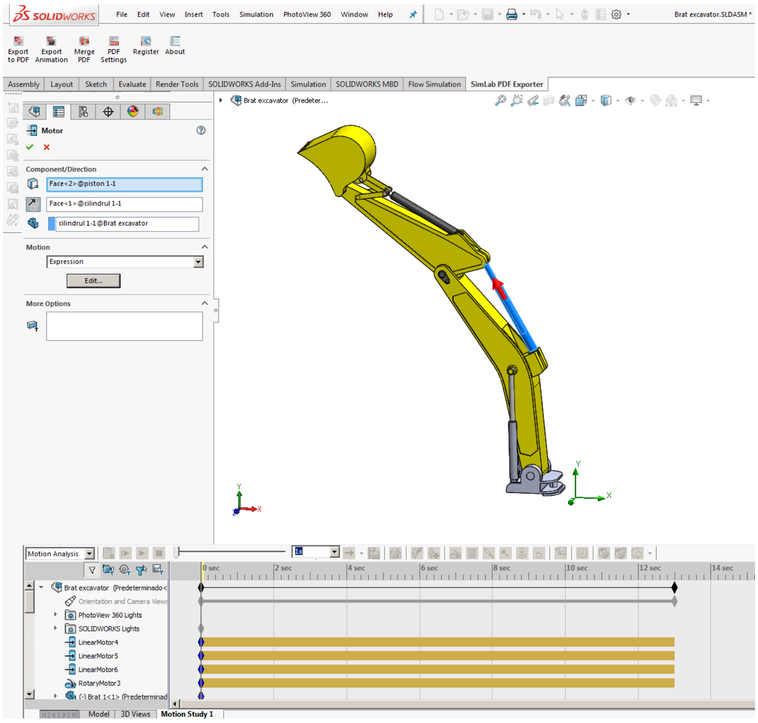

Fig. 7. Attachment of the linear drive motor of the hydraulic cylinder for the motion of the arm itself

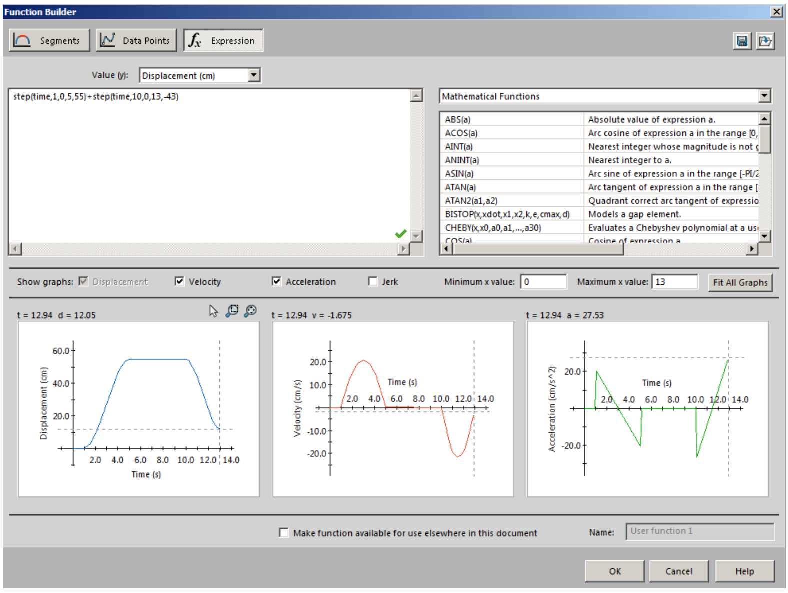

Fig. 8. Menu for implementing the mathematical expression describing the motion of the hydraulic cylinder for the motion of the arm itself

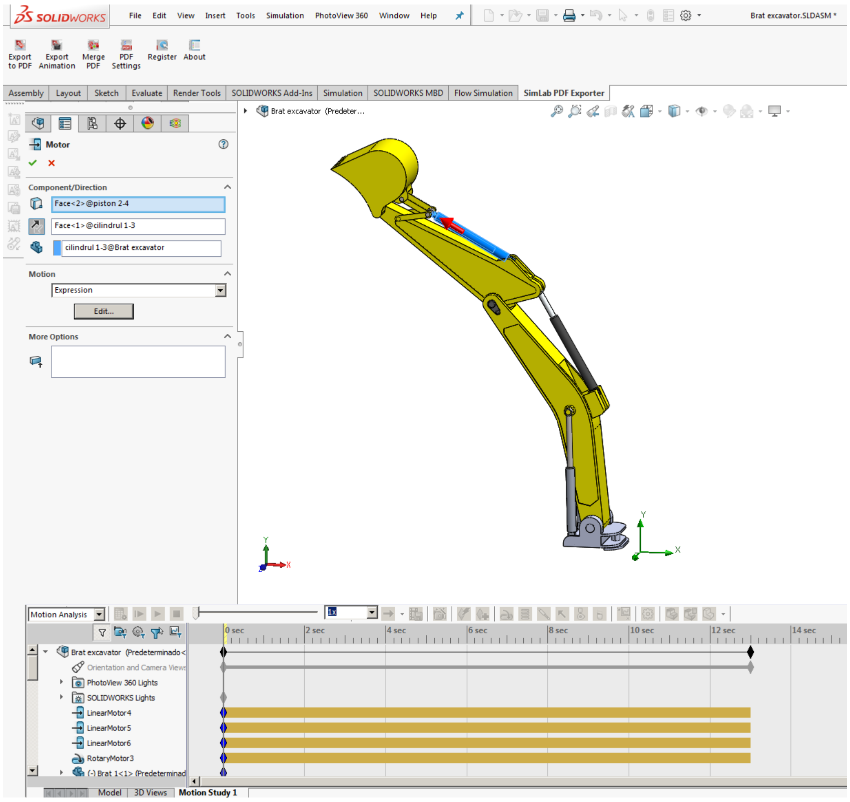

Fig. 9. Attachment of the linear drive motor of the hydraulic cylinder for the motion of the excavation bucket

The actuation of the excavation bucket is ensured, as shown in Figure 9, also by a linear motor that determines the motion of the piston rod end relative to the base of the hydraulic cylinder. As in the case of the other two linear motors, the motion equation is described using the `STEP` function.

Thus, after implementing relation (16), the graph of the displacement variation graphs of velocity and acceleration (Figure 10). In this case, the `STEP` function ensures smooth transition of the piston displacement variation curve for the considered time intervals: (3,5), (4.5, 5.5), and (10,13) seconds.

The first two-time intervals correspond to the excavation process and to the positioning of the bucket in order to perform the pivoting motion of the mini excavator arm. The last time interval corresponds to the unloading of the excavator bucket after the completion of the pivoting motion.

$$\text{step}\left(time,3,0,5,20\right)+\text{step}\left(time,4.5,0,5.5,20\right)+\text{step}\left(time,10,0,13,\text{-}32\right) \quad (16)$$

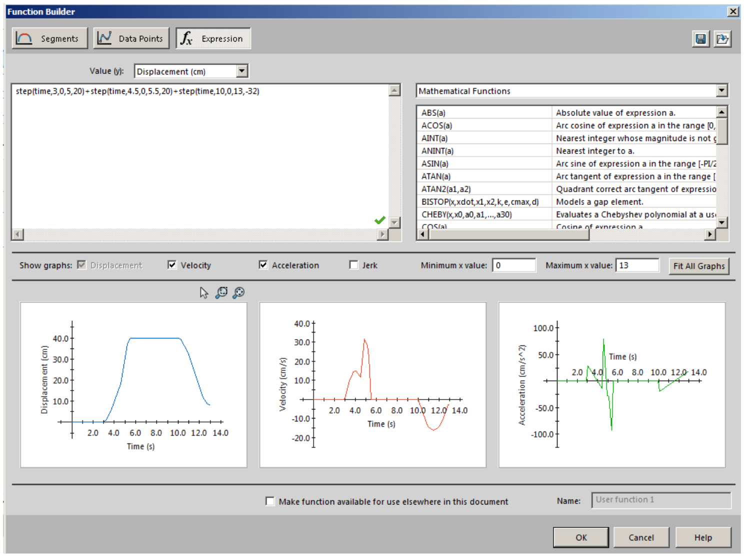

Fig. 10. Menu for implementing the mathematical expression describing the motion of the hydraulic cylinder for the motion of the excavation bucket

The pivoting motion of the mini excavator arm is achieved with the aid of a rotary motor, as shown in Figure 11. The motor performs a rotation at a constant angular velocity through an angle of 50 degrees, starting from the 7th second up to the 10th second of the simulation.

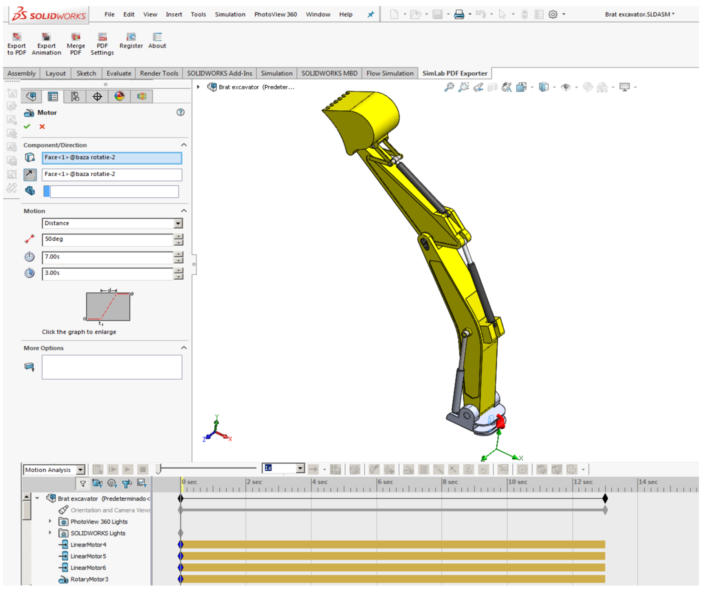

Fig. 11. Attachment of the rotary motor that produces the pivoting motion of the arm

5. TECHNOLOGICAL VOLUME EVALUATION

Following the simulation of the bucket lowering, cutting, lifting, transport, and unloading processes of the excavated material, a first result obtained is the trajectory of a tooth on the bucket, as shown in Figure 12. It should be emphasized that this trajectory is the result of a specific excavation scheme and is not unique.

The length of the trajectory described by the excavation bucket tooth is 10.770,30 mm. The determination of the technological volume (Figure 13) that encloses the bucket trajectory (in the analyzed case, the trajectory of a tooth on the bucket) was carried out using two methods.

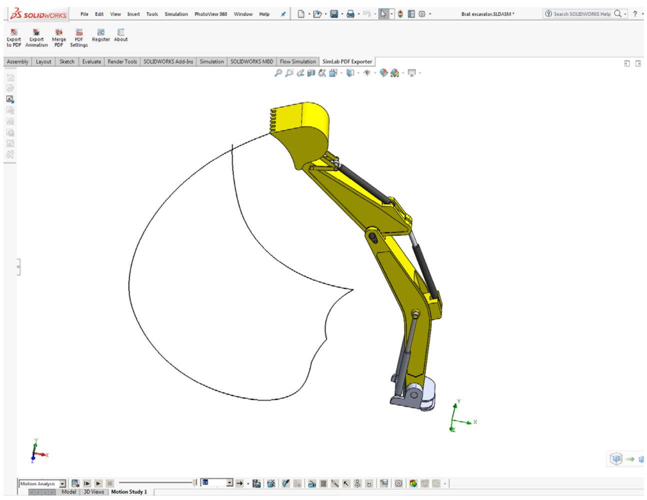

Fig. 12. Trajectory of a tooth of the mini excavator bucket

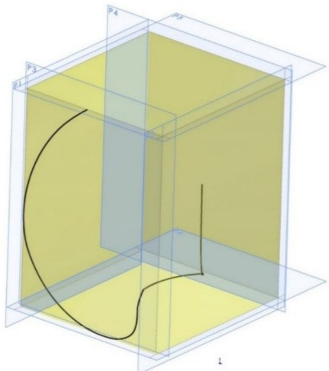

Fig. 13. Technological volume of the excavation process

The first calculation method is based on the capability of the SOLIDWORKS application to export the coordinates x_i, y_i and z_i of the points corresponding to a 3D trajectory into a `.CSV` file format that is compatible with Microsoft Excel.

Thus, using the `MIN` and `MAX` mathematical functions of the Excel application, the limits within the coordinates vary for the three directions, X, Y and Z were determined, as can be observed in table 1. Using these values, the lengths of the edges of the rectangular parallelepiped representing the technological volume of the excavation process were determined.

Table 1. **Determination of the dimensions of the edges of technological volume**

Limits in *X* direction [mm]		Limits in *Y* direction [mm]		Limits in *Z* direction [mm]	
Min	Max	Min	Max	Min	Max
-4075	-1355	-293	3422	-3103	-101
Edge length *X* [mm]		Edge length *Y* [mm]		Edge length *Z* [mm]	
2721		3716		3003	

Using the values from Table 1, it follows that the technological volume of excavation process is $V_{TH} = 30{,}35\ \text{m}^3$.

The second method for calculating the technological volume is based on the construction of parallelepiped whose edges are determined by the intersection of planes $P_1, \ldots, P_2$, which bound the trajectory of the bucket tooth (Figure 14)

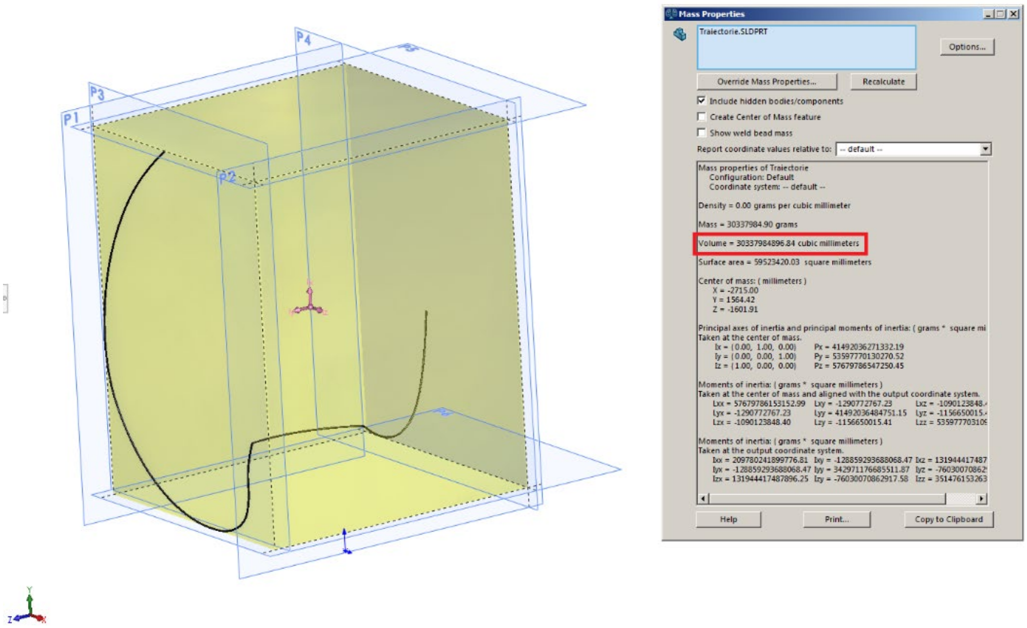

Fig. 14. Construction of the technological volume at the intersection of the planes bounding the trajectory of the bucket tooth

The SOLIDWORKS application allows the evaluation of volume, mass, as well as other geometric and mechanical properties for the 3D entities constructed using it. As can be seen in this case the calculated technological volume is 30.33 m^3. If the results obtained by the two presented calculation methods are compared, the percentage difference is approximately 0,066%. Thus, it can be concluded that the obtained results are satisfactory.

It should be noted that the value of a technological volume depends on the constructive geometric data of the mini excavator arm as well as on its operating conditions. The technological volume previously determined using the two calculation methods is specific to the analyzed mini excavator for certain excavation parameters.

CONCLUSIONS

Excavators with boom and a single bucket are mechano-hydraulic-electrical machines used for the excavation and loading of ore or rock-type materials in mining and construction application.

The performance of these machines, such as the working space, technological forces, cutting capacity, and loading capacity, is closely related to the structure and operation, as well as to the kinematics and dynamics of the bucket-boom mechanism.

Since the mechanism is, from a structural point of view, an open planar kinematic chain, with the kinematic elements – namely the arm and the bucket – connected by kinematic joints, usually actuated by hydraulic cylinders, the kinematic, kineto-static, and dynamic study of these machines must rely on dedicated methods known from the literature.

In order to determine the functional technological parameters imposed by the working technology, these analytical-theoretical methods must be implemented within a computer-based modeling and simulation system.

For the application and validation of the presented model, an existing excavator was selected, whose geometric and kinematic characteristics were available in the technical documentation. For this purpose, a virtual model of the boom of this excavator was developed in the SOLIDWORKS application. The boom model subjected to analysis was created as an assembly consisting of parts between which geometric relationship of coincidence, concentricity, and distance were established. Subsequently, the characteristics of the parts composing the boom assembly and their implementation were presented.

On this basis, the simulation of the kinematics of the excavation-loading mechanism, namely the bucket-boom mechanism of the selected excavator, was carried out using the developed model. The study was performed in SOLIDWORKS application, using the `Motion Analysis` study type from the `Motion Study` menu.

This menu provides the capability to assign rotary or linear actuators (motors) to an assembly, for which the motion equations can be expressed in terms of constant velocities, distances traveled over specified time intervals, periodic oscillations, trajectory segments, points, as well as mathematical expressions.

In order to highlight as clearly as possible, the time sequence of the action of the four actuators that define the kinematics of the excavator boom, a Gantt-type diagram was plotted, followed by graphical representations of the variation of the distance traveled by the piston of each hydraulic actuation cylinder.

A first result obtained from the simulation of the excavation process, including the sequence of phases consisting of bucket lowering, cutting, lifting, transport, and unloading of the excavated material, is the determination of the spatial trajectory of a tooth on the excavator bucket for a specific excavation scheme.

Based on the developed model, the magnitude of the technological volume (referred to as the working space in manipulator theory), namely the volume enclosing the bucket trajectory (in the analyzed case, the trajectory of a tooth on the bucket), was determined by simulation using two different methods. The percentage difference between the obtained results is approximately 0.066%.

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