GRAPHIC AND ANALYTICAL DETERMINATION OF THE CENTER OF MASS OF A HOMOGENEOUS L-SHAPED PLATE

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Abstract: This paper addresses the determination of the center of mass of a flat homogeneous plate with an irregular geometry, specifically an L-shaped configuration, using a graphic method supported by analytical validation. The proposed approach relies on decomposing the plate into simpler rectangular components whose centers of gravity can be easily identified. By applying the graphic construction in two distinct decomposition schemes and intersecting the resulting lines that connect partial centers of mass, a unique position of the global center of mass is obtained. To confirm the accuracy and consistency of the graphic method, an analytical demonstration based on classical mechanics equations for systems of parallel forces is provided. The study proves that, regardless of the chosen decomposition, the resulting center of mass remains identical. The method offers a practical and intuitive alternative for determining the center of gravity of homogeneous flat bodies with complex shapes and can be effectively applied in engineering education and preliminary design analyses.

Key words: center of mass; center of gravity; homogeneous plate; graphic method; analytical validation.

1. INTRODUCTION

The existence of the gravity field of the Earth leads to exerting an interaction from a distance between a body and the Earth, namely, the force of gravitational attraction. Gravitational forces represent the classic case of parallel force. These forces can be reduced to a resultant applied in the center of parallel forces, which in this case becomes the center of gravity. The phrase center of gravity is often interchangeable with center of masses, but they are different physical concepts. The two centers overlap in a uniform

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gravitational field.

The center of mass represents a concept of a more general character than the center of gravity, being defined independently compared to the latter. In the absence of the gravitational field, the center of gravity loses its sense, while the center of mass continues to exist. Similarly, in an uneven gravitational field, the two points (centers) occupy different positions. It is noted that in the context of current technical problems, one can consider that the positions of the two coincide.

The position of the center of gravity of a body is determined more easily when the latter is homogeneous and has symmetry elements[1].

Determination of the center of gravity of a compound body can be done with the help of partial centers of gravity, the body having a shape allowing it being divided in several component parts, for each of the parts the mass (weight) and the position of the center of gravity being known.

The center of gravity of a homogeneous body and of regular geometric shape depends on the geometric elements of the body, not on the nature of the material of which it is made, and can be decomposed in component parts, also of regular geometric shape. If these bodies have plane, an axis or a symmetry center, then the center of gravity is found in the respective plane, on the axis or in the center of symmetry, respectively[2,3].

The center of gravity of the simple homogeneous bodies, of regular geometric shapes (for example, square, rectangle, triangle) can be determined graphically (geometrically) as well.

We shall further present aspects on the graphic method applied in the determination of the center of gravity of a homogeneous flat plane of given configuration and the justification of this method.

2. GRAPHIC DETERMINATION OF THE CENTER OF MASS OF THE PLATE

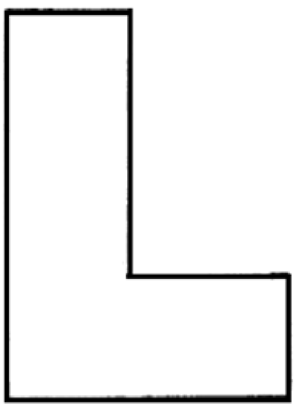

Fig. 1. Geometric configuration of the homogeneous L-shaped plate

We consider the homogeneous plate having the given configuration (in the shape of the letter L), in figure 1, and we aim to determine the center of weight of the plate, with the help of a ruler and a sharp pencil. As it is seen from the figure, the plate has straight angles, and it is also homogeneous and has constant thickness.

We shall further proceed to explain the procedure used, showing how the center of gravity of the plate has been localized.

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The solving follows. As it is noticed, the plate in the shape of the letter *L* is formed by joining two rectangular plates, of unequal lengths.

Their centers of gravity are found at the intersection of diagonals (in the very middle of the respective rectangles). The center of the plate in the form of the letter *L* is found on the line that joins the centers of the two rectangular plates.

The arms of the letter *L* are not equal however. therefore, the choice of the two rectangular joined plates can be done in two distinctive ways (see the dotted line in the drawing in figure 2).

The first way is formed by the rectangular plates having their centers of gravity C_1 and C_2, respectively.

The second way is formed of rectangular plates having their centers of gravity C_3 and C_4, respectively.

Point *C* (of intersection of the two straight lines obtained by joining the centers of gravity of the selected plates in the two ways) is the center of gravity of the plate in the shape of the letter *L*.

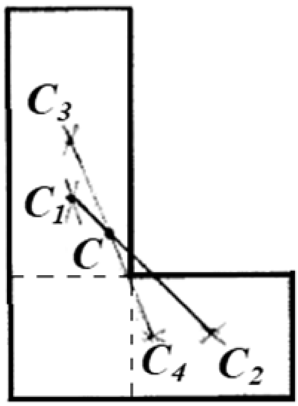


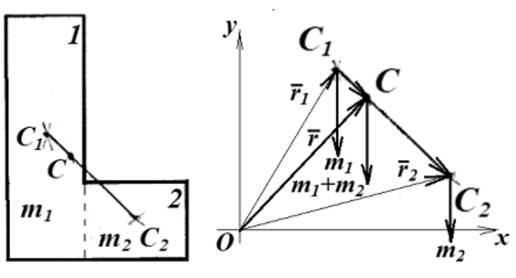
Fig. 2. Graphic determination of the center of mass

3. THE ANALYTICAL SOLUTION FOR THE APPLIED GRAPHIC METHOD

In order to justify the graphic way in which the position of the center of gravity of the homogeneous flat plate has been obtained, we shall demonstrate its reliability be analytical method[4].

We shall consider the first way of selecting the component plates as in Fig. 3 a with centers of gravity, rectangular plate 1, of mass m_1 and the center of gravity in point C_1, and the rectangular plate 2, respectively, of mass m_2 and center of gravity in point C_2.

Let us have in plane *xOy* (Fig. 3.b.) point C_1 of mass m_1 and its vector of position $\overline{r_1} = x_1\overline{i} + y_1\overline{j}$, and point C_2, respectively, of mass m_2 and its vector of position $\overline{r_2} = x_2\overline{i} + y_2\overline{j}$, center of mass (weight) *C*, of mass m_1+m_2 of the wo points, having as vector of position $\overline{r} = x_C\overline{i} + y_C\overline{j}$ (according to the equations to determine the center of mass in mechanics) has the coordinates given by the equations:



A B

Fig.3. A) First plate decomposition
B) Cartesian model of the first decomposition

$$x_C = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \quad y_C = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}, \tag{1}$$

We verify if the center of mass *C* is on the line determined by points C_1 and C_2. For this, we use the condition that has to be met by the extremities of three vectors that these extremities should be colinear.

We consider the vectors of position of the material points C_1, C_2 and $\overline{OC_1}$, $\overline{OC_2}$, respectively, and the vector of position of the resultant center of mass *C*, and $\overline{OC}$, respectively.

The condition for the extremity of the vector $\overline{OC} = \overline{r}$ to be found on line C_1C_2 is that in the equation of dependence:

$$\overline{OC} = \lambda_1 \overline{OC_1} + \lambda_2 \overline{OC_2} \quad sau \quad \overline{r} = \lambda_1 \overline{r_1} + \lambda_2 \overline{r_2}, \tag{2}$$

to have $\lambda_1 + \lambda_2 = 1$.

We write:

$$\overline{r} = \lambda_1 \overline{r_1} + \lambda_2 \overline{r_2} \Leftrightarrow x_C \overline{i} + y_C \overline{j} = \lambda_1 \left(x_1 \overline{i} + y_1 \overline{j} \right) + \lambda_2 \left(x_2 \overline{i} + y_2 \overline{j} \right), \tag{3}$$

whence considering equation (1) and the expressions of vectors $\overline{r_1}$ and $\overline{r_2}$, we have:

$$\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \overline{i} = \left(\lambda_1 x_1 + \lambda_2 x_2 \right) \overline{i}, \quad \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \overline{j} = \left(\lambda_1 y_1 + \lambda_2 y_2 \right) \overline{j}, \tag{4}$$

Thus from (4) results:

$$\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \lambda_1 x_1 + \lambda_2 x_2 \Leftrightarrow m_1 x_1 + m_2 x_2 = \lambda_1 x_1 \left(m_1 + m_2 \right) + \lambda_2 x_2 \left(m_1 + m_2 \right), \tag{5}$$

Identifying term by term in equation (5) we get:

$$m_1 = \lambda_1 \left(m_1 + m_2 \right) \Rightarrow \lambda_1 = \frac{m_1}{m_1 + m_2}, \quad m_2 = \lambda_2 \left(m_1 + m_2 \right) \Rightarrow \lambda_2 = \frac{m_2}{m_1 + m_2}, \tag{6}$$

whence:

$$\lambda_1 + \lambda_2 = \frac{m_1}{m_1 + m_2} + \frac{m_2}{m_1 + m_2} = \frac{m_1 + m_2}{m_1 + m_2} = 1, \tag{7}$$

In the end it results that points C_1, C_2 and *C* are colinear.

We verify whether the center of gravity C is found between points C_1 and C_2. Thus *C* is inside the segment $[C_1C_2]$, $C_1(x_1,y_1)$, $C_2(x_2,y_2)$ if $C_1C_2=C_1C+CC_2$.

That is, vectorially one writes that if: $\overline{C_1C_2} = \overline{C_1C} + \overline{CC_2}$, then $C \in \overline{C_1C_2}$. From Fig. 4 it results:

$$\left.\begin{array}{l}\overline{r}=\overline{r}_1+\overline{C_1C}\Rightarrow\overline{r}_1=\overline{r}-\overline{C_1C}\\ \overline{r}_2=\overline{r}+\overline{CC_2}\\ \overline{r}_2=\overline{r}_1+\overline{C_1C_2}\Rightarrow\overline{C_1C_2}=\overline{r}_2-\overline{r}_1\end{array}\right\}\Rightarrow\begin{cases}\overline{C_1C_2}=\overline{r}_2-\overline{r}_1=\overline{r}+\overline{CC_2}-\left(\overline{r}-\overline{C_1C}\right)=\\ =\overline{r}+\overline{CC_2}-\overline{r}+\overline{C_1C}=\overline{C_1C}+\overline{CC_2}\end{cases},\quad(8)$$

We shall further verify by another method whether vectors $\overline{C_1C}$ and $\overline{C_1C_2}$ are colinear ($C\in\overline{C_1C_2}$) and whether the center of gravity C is found between points C_1 and C_2. Thus from Fig. 3B it results:

$$\begin{array}{l}\overline{OC_1}+\overline{C_1C}=\overline{OC}\quad\Rightarrow\quad\overline{C_1C}=\overline{OC}-\overline{OC_1}\\ \overline{OC_1}+\overline{C_1C_2}=\overline{OC_2}\quad\Rightarrow\quad\overline{C_1C_2}=\overline{OC_2}-\overline{OC_1}\end{array},\quad(9)$$

where:

$$\overline{OC_1}=\overline{r}_1=\overline{x_1\overline{i}+y_1\overline{j}},\ \overline{OC_2}=\overline{r}_2=x_2\overline{i}+y_2\overline{j},\ \ \overline{OC}=\overline{r}=x_C\overline{i}+y_C\overline{j}\ ,\quad(10)$$

hence:

$$\begin{array}{l}\overline{C_1C_2}=\left(x_2-x_1\right)\overline{i}+\left(y_2-y_1\right)\overline{j}\\ \overline{C_1C}=\left(\dfrac{m_1x_1+m_2x_2}{m_1+m_2}-x_1\right)\overline{i}+\left(\dfrac{m_1y_1+m_2y_2}{m_1+m_2}-y_1\right)\overline{j}=\\ =\left(\dfrac{\cancel{m_1x_1}+m_2x_2-\cancel{m_1x_1}-m_2x_1}{m_1+m_2}\right)\overline{i}+\left(\dfrac{\cancel{m_1y_1}+m_2y_2-\overline{m_1y_1}+m_2y_1}{m_1+m_2}\right)\overline{j}=\\ =\dfrac{m_2}{m_1+m_2}\left[\left(x_2-x_1\right)\overline{i}+\left(y_2-y_1\right)\overline{j}\right]=\dfrac{m_2}{m_1+m_2}\overline{C_1C_2}\end{array},\quad(11)$$

Thus:

$$\overline{C_1C}=\mu_1\overline{C_1C_2},\ \ \left(\mu_1=\frac{m_2}{m_1+m_2}\lhd 1\right),\quad(12)$$

In conclusion, vectors $\overline{C_1C_2}$ and $\overline{C_1C}$ are colinear ($\overline{C_1C}=\mu_1\overline{C_1C_2}$ and vectors having the same origin) and the center of gravity C is found between points C_1 and C_2, $\mu_1 < 1$.

We shall further consider the second way of choosing the component plates, as in Fig. 5, with the centers of weight, rectangular plate *3* of mass m_3=m_1-m and the center of weight in point C_3 and the rectangular plate *4,* respectively, of mass m_4=m_2+m and the center of weight in point C_4.

Let it be now in plane *xOy* point C_3 of mass $m_3=m_1-m$ and vector of position $\overline{r_3} = x_3\overline{i} + y_3\overline{j}$ and point C_4, respectively masa $m_4=m_2+m$ and vector of position $\overline{r_4} = x_4\overline{i} + y_4\overline{j}$ center of mass C' of the two points having as vector of position $\overline{r'} = x_{C'}\overline{i} + y_{C'}\overline{j}$ has the coordinates (according to the equations for determination of the center of mass in mechanics), given by the equations:

$$\begin{aligned} x_{C'} &= \frac{m_3x_3 + m_4x_4}{m_3 + m_4} = \frac{(m_1 - m)x_3 + (m_2 + m)x_4}{m_1 - m + m_2 + m} = \frac{x_3m_1 + x_4m_2 + m(x_4 - x_3)}{m_1 + m_2} \\ y_{C'} &= \frac{m_3y_3 + m_4y_4}{m_3 + m_4}\frac{(m_1 - m)y_3 + (m_2 + m)y_4}{m_1 - m + m_2 + m} = \frac{y_3m_1 + y_4m_2 + m(y_4 - y_3)}{m_1 + m_2} \end{aligned} \tag{13}$$

The center of gravity C' should coincide with the center of mass C ($C' \equiv C$), since the same flat plane cannot have two centers of mass (weight), that is ($\overline{OC'} \equiv \overline{OC}$). By partitioning the plate in the two ways, the mass of the plate stays the same.

We shall further verify whether vectors $\overline{C_3C_4}$ and $\overline{C_3C'} \equiv \overline{C_3C}$ are colinear ($C' \equiv C \in \overline{C_3C_4}$) and whether the center of gravity C' (that is, C) is found between points C_3 and C_4. We thus have according to Figure 2:

$$\begin{aligned} &\overline{OC_3} + \overline{C_3C} = \overline{OC} \;\; \Rightarrow \;\; \overline{C_3C} = \overline{OC} - \overline{OC_3} \\ &\overline{OC_3} + \overline{C_3C_4} = \overline{OC_4} \;\; \Rightarrow \;\; \overline{C_3C_4} = \overline{OC_4} - \overline{OC_3} \end{aligned}, \tag{14}$$

where:

$$\begin{aligned} &\overline{OC_3} = \overline{r_3} = \overline{x_3'\overline{i} + y_3'\overline{j}} \\ &\overline{OC_4} = \overline{r_4} = x_4\overline{i} + y_4 j \\ &\overline{OC'} \equiv \overline{OC} = \overline{r} = x_C\overline{i} + y_C\overline{j} \end{aligned}, \tag{15}$$

hence:

$$\begin{aligned} &\overline{C_3C_4} = (x_4 - x_3)\overline{i} + (y_4 - y_3)\overline{j} \\ &\overline{C_3C} = \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2} - x_3\right)\overline{i} + \left(\frac{m_1y_1 + m_2y_2}{m_1 + m_2} - y_3\right)\overline{j} \end{aligned}, \tag{16}$$

But, since $C' = C$, results:

$$\frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{x_3m_1 + x_4m_2 + m(x_4 - x_3)}{m_1 + m_2} \tag{17}$$

and

$$\frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{y_3 m_1 + y_4 m_2 + m\left(y_4 - y_3\right)}{m_1 + m_2} \tag{18}$$

Considering (17) and (18) we can write:

$$\begin{aligned}
\overline{C_3C} &= \left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} - x_3\right)\overline{i} + \left(\frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} - y_3\right)\overline{j} = \\
&= \left[\frac{x_3 m_1 + x_4 m_2 + m\left(x_4 - x_3\right)}{m_1 + m_2} - x_3\right]\overline{i} + \left[\frac{y_3 m_1 + y_4 m_2 + m\left(y_4 - y_3\right)}{m_1 + m_2} - y_3\right]\overline{j} = \\
&= \left[\frac{x_3 m_1 + x_4 m_2 + m\left(x_4 - x_3\right) - x_3\left(m_1 + m_2\right)}{m_1 + m_2}\right]\overline{i} + \\
&+ \left[\frac{y_3 m_1 + y_4 m_2 + m\left(y_4 - y_3\right) - y_3\left(m_1 + m_2\right)}{m_1 + m_2}\right]\overline{j} = \\
&= \frac{1}{m_1 + m_2}\left[\left(m_2 + m\right)\left(x_4 - x_3\right)\overline{i} + \left(m_2 + m\right)\left(y_4 - y_3\right)\overline{j}\right] = \\
&= \frac{m_2 + m}{m_1 + m_2}\overline{C_3C_4}
\end{aligned}, \tag{19}$$

Thus:

$$\overline{C_3C} = \mu_2 \overline{C_3C_4}, \quad \left(\mu_2 = \frac{m_2 + m}{m_1 + m_2} \lhd 1\right), \tag{20}$$

In conclusion, vectors $\overline{C_3C_4}$ și $\overline{C_3C}$ are colinear ($\overline{C_3C} = \mu_2 \overline{C_3C_4}$ vectors having the same origin) and the center of gravity ($C' \equiv C$) is found between C_3 and C_4, $\mu_2 < 1$.

Thus, in the end, the center of gravity of the plate is found at the intersection of the lines that pass through points C_1, C_2 (line $\overline{C_1C_2}$), and C_3, C_4 (line $\overline{C_3C_4}$), respectively.

That is, $C = \overline{C_1C_2} \cap \overline{C_3C_4}$.

As it has been noted, the plate in the form of the letter L is formed by joining two rectangular plates, of unequal lengths. Thus, we consider lengths L_1 and L_2, respectively, and widths equal with l for both rectangles.

We shall determine with these dimensions the coordinates of the center of mass for the first mode of choosing the placement of the plates. (Fig. 4).

Let's have plate *1* with $C_1\left(\frac{l}{2},\frac{L_1}{2}\right)$ and $A_1 = L_1 \cdot l$ and plate *2* with $C_2\left(\frac{L_2}{2}+l,\frac{L_1}{2}\right)$ and $A_2 = L_2 \cdot l$ a=then according to the equations in mechanics, the coordinates of the center of mass of the plate in the shape of *L* are given by the relationships:

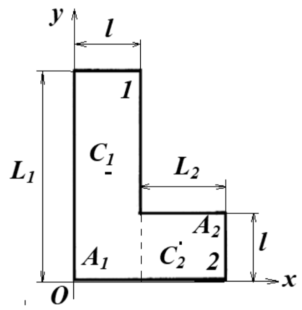


Fig. 4. Geometric dimensions and center of mass coordinates for the first decomposition scheme

$$x_C = \frac{\frac{l}{2}L_1 l + \left(\frac{L_2}{2}+l\right)L_2 l}{L_1 l + L_2 l} = \frac{l\left(L_1+2L_2\right)+L_2^2}{2\left(L_1+L_2\right)}$$

$$y_C = \frac{\frac{L_1}{2}L_1 l + \frac{l}{2}L_2 l}{L_1 l + L_2 l} = \frac{L_1^2 + L_2 l}{2\left(L_1+L_2\right)} \qquad (21)$$

We shall further determine with these dimensions the coordinates of the center of mass for the second way of selection of the placing of the plates (Fig.5).

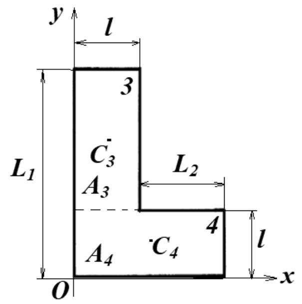


Fig. 5. Geometric dimensions and center of mass coordinates for the second decomposition scheme

Let's have plate *3* with the center of mass $C_3\left(\frac{l}{2},\frac{L_1-l}{2}+l\right)$ and area $A_3 = l\left(L_1-l\right)$ and plate *4* with the center of mass $C_4\left(\frac{L_2+l}{2},\frac{L_1}{2}\right)$ and area $A_4 = l\left(L_2+l\right)$, similarly according to the equations in mechanics, the coordinates of the center of mass of the plate in the shape of *L* are given by equations:

$$x'_C = \frac{\frac{l}{2}\left(L_1-l\right)l + \left(\frac{L_2+l}{2}\right)\left(L_2+l\right)l}{l\left(L_1-l+L_2+l\right)} = \frac{l\left(L_1+2L_2\right)+L_2^2}{2\left(L_1+L_2\right)},$$

$$y'_C = \frac{\frac{L_1+l}{2}\left(L_1-l\right)l + \frac{l}{2}\left(L_2+l\right)l}{l\left(L_1-l+L_2+l\right)} = \frac{L_1^2 + L_2 l}{2\left(L_1+L_2\right)} \qquad (22)$$

It is noted that in both cases, the coordinates of the center of mass are the same.

Another analytical method by which it can be demonstrated that the center of mass is the same in both cases, is to determine the point of intersection of the equations of lines determined by two points, these points being the very centers of mass of the plates in the two ways of placement.

Thus, the equations of the lines determined by C_1 and C_2 (Fig. 4) and C_3 and C_4 (Fig. 5) are given by equations:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}\Leftrightarrow\frac{y-\frac{L_1}{2}}{\frac{l}{2}-\frac{L_1}{2}}=\frac{x-\frac{l}{2}}{\left(\frac{L_2}{2}+l\right)-\frac{l}{2}}\Rightarrow,$$
$$y=\frac{2\left(l-L_1\right)x-l^2+L_1L_2+2lL_1}{2\left(L_2+l\right)} \tag{23}$$

$$\frac{y-y_3}{y_4-y_3}=\frac{x-x_3}{x_4-x_3}\Leftrightarrow\frac{y-\frac{L_1+l}{2}}{\frac{l}{2}-\frac{L_1+l}{2}}=\frac{x-\frac{l}{2}}{\frac{L_2+l}{2}-\frac{l}{2}}\Rightarrow,$$
$$y=\frac{-2L_1x+l\left(L_1+L_2\right)+L_1L_2}{2L_2} \tag{24}$$

which intersected give the same values (coordinates) for the center of mass of the flat plate.

4. CONCLUSIONS

The present paper analyzed a graphic method for determining the center of mass of a homogeneous flat plate with a composite geometry, specifically an L-shaped configuration, highlighting both the practical applicability of the method and its theoretical foundation through analytical demonstration. The study confirms that graphic methods, although seemingly simple, can lead to rigorous and accurate results when applied on solid mechanical principles.

One of the main outcomes of this work is the demonstration that the center of mass of a homogeneous flat plate is unique and independent of the manner in which the body is decomposed into component elements. In the analyzed case, the plate was divided into rectangular components in two distinct ways, each leading to different partial centers of mass. Nevertheless, the intersection of the lines connecting these partial centers consistently yields the same point, which represents the center of mass of the entire plate. This result confirms the consistency and correctness of the applied graphic method.

The analytical justification of the graphic construction represents another essential contribution of the paper. By employing classical relations from theoretical mechanics for determining the center of mass of a system of material points, it was demonstrated that the point obtained graphically lies on the segment connecting the centers of mass of the components and that its position is governed by the ratio of their masses (areas). The analytical verification confirms that, in this case, the graphic method is not approximate, but exactly reflects the mathematical solution of the problem.

The presented method proves to be particularly useful in the analysis of homogeneous planar bodies with irregular or composite shapes, for which direct determination of the center of mass through integration may be difficult or time-consuming. By decomposing the body into simple geometric figures and using elementary geometric constructions, the position of the center of mass can be identified rapidly and clearly. This approach is advantageous both in preliminary stages of engineering design and in verification or quick estimation tasks.

Furthermore, the paper emphasizes the educational value of the graphic method. It facilitates a clearer understanding of the concept of center of mass and of the relationship between geometry and mass distribution, offering students an intuitive visual representation of a fundamental concept in mechanics. The correlation between the graphic construction and the analytical demonstration contributes to the development of logical reasoning and to a deeper understanding of classical mechanics principles.

In conclusion, the graphic method for determining the center of mass analyzed in this paper is accurate, rigorous, and easy to apply for homogeneous flat plates with composite geometries. The analytical validation demonstrates that the obtained results are independent of the chosen decomposition scheme, confirming the general character of the method.

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