MINIMIZING THE ACTUATING POWER OF MULTI-ROPE HOISTING MACHINERY

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Abstract: The specific energy consumption is mainly influenced by kinematics and dynamic measures of vertical transport installations as well as by the compatibility of different composing parts and their subcomponents. The optimization of kinematics and dynamic parameters characterizing a transport cycle is decisive considering the energy consumption. Also considering the operation of the vertical transport installations, as well as the character of the variation of kinematics and dynamic parameters during a race, it has been considered that one of the adequate optimization methods of these parameters is the calculus of variations. In order to apply this calculus, the definition of the optimization functional and restrictions is imposed. One of the basic performance parameters of the operation of the vertical transport installations is the specific energy consumption during a cycle. It therefore means that the optimization of the transport cycle related to this parameter may be realized using a functional with a function under the integral depending on the electric energy consumption during a race.

Keywords: optimization; unbalanced installations; statically balanced installation; dynamically balanced.

1. GENERAL CONSIDERATIONS REGARDING THE CALCULUS OF VARIATIONS

The purpose of the calculus of variations is the discovery of functions which may reach extreme values (either maximum or minimum), for some measures depending on them also called functional. The functional may be considered a generalization of the analysis of some functions of a certain type in which the role of the variable being played by another function, such as:

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$$\exists = \int_a^b f(x, y, y')dx \tag{1}$$

Namely the defined integral of expression *f* which depends on the independent variable *x*, the searched function *y(x)* and its derivative *y′*. While functional (1) using the following sum:

$$\exists = \sum_{i=1}^{n} f\left(x_i ; y_i ; \frac{y_i - y_{i-1}}{\Delta x} \right) \Delta x \tag{2}$$

The issue is therefore the determination of the extreme of the function $Y(y_1, y_2, ..., y_n)$ of more variables. Higher as n will be as precise the approximation will be, getting closer to solving the problem of variations. If *y(x)* represents the extreme of functional (1), then the following is necessary:

$$f_y - \frac{d}{dt} f_{y1} = 0 \tag{3}$$

Relation also called Euler's equation. In the calculation of variations, functions are seldom met, which depend not only on the first one but also on the superior order derivatives of the determined function.

The aspects of these functions are:

$$\exists = \int_{x_0}^{x_1} f(x, y, y', y'', ..., y^{(n)})dx \tag{4}$$

Therefore, the curve reaching an extreme value needs to satisfy the following equation:

$$f_y - \frac{d}{dx} f_{y'} + \frac{d^2}{dx^2} f_{y''} - ... + (-1)^n \frac{d^n}{dx^n} f_{y^{(n)}} = 0 \tag{5}$$

For the case where functional (5) depends only on the first and second order derivatives and has the following form:

$$\exists = \int_{x_0}^{x_1} f(x, y, y', y'')dx \tag{6}$$

Function $y = y(x)$ which realizes the extreme of functional ∃ is determined, and y and y' are given in $x = x_0$ and $x = x_1$. Examining the effect on functional ∃ of a variation *a* of *y* with a small quantity i which for function i with its fixed ends satisfies the condition $\delta y = \delta y' = 0$ when $x = x_0$ and $x = x_1$, the first variation of functional ∃ is:

$$\delta \exists = \int_{x_0}^{x_1} \left(\frac{\partial f}{\partial y} \delta y + \frac{\partial f}{\partial y'} \delta y' + \frac{\partial f}{\partial y''} \delta y'' \right) dx \tag{7}$$

If function $\exists$ takes an extreme value, then expression (7) becomes null. Integrating in parts the second and the third term in expression (7) in order to eliminate variations $\delta y'$ and $\delta y''$, the following is obtained:

$$\delta \exists = \int_{x_0}^{x_1} \left(\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} + \frac{d^2}{dx^2} \frac{\partial f}{\partial y''} \right) \delta y \, dx + \left[\left(\frac{\partial f}{\partial y'} - \frac{d}{dx} \frac{\partial f}{\partial y''} \delta y \right) \right]_{x_0}^{x_1} + \left[\frac{\partial f}{\partial y''} \delta y' \right]_{x_0}^{x_1} = 0 \tag{8}$$

If the limit conditions are imposed $\delta y = \delta y' = 0$ for $x = x_0$ and $x = x_1$, then

$$\delta \exists = \int_{0}^{x_1} \left(\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} + \frac{d^2}{dx^2} \frac{\partial f}{\partial y''} \right) \delta y \, dx = 0 \tag{9}$$

The above integral needs to be cancelled for all the admissible δy values, imposing that the expression in the parenthesis of relation (9) becomes zero:

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) + \frac{d^2}{dx^2} \left(\frac{\partial f}{\partial y''} \right) = 0 \tag{10}$$

This expression is also known as Euler - Poisson’s equation. It is a fourth-degree differential equation, the solving of which gives extreme values of function $\exists$.

2. ESTABLISHING THE OPTIMISATION FUNCTIONAL FOR SINGLE CABLE VERTICAL TRANSPORT INSTALLATIONS POWERED BY AN ASYNCHRONOUS MOTOR

The actual peripheral force is:

$$F_{ef} = \sqrt{\frac{\int_0^T F^2 dt}{T_{ef}}} \approx \sqrt{\frac{\sum F_i^2 t_i}{T_{ef}}} \; [N] \tag{11}$$

Because function F(t) varies during different phases, the integral $\int_0^T F^2 dt$ is solved separately for each phase:

$$\int_0^T F^2 dt = \sum_0^n \int_0^{ti} F_i^2 dt \tag{12}$$

According to the general equation of the dynamics of vertical transport installations, the force at periphery of the reeling organism is expressed using the following relation:

$$F = \left[k\,Q_u + (q - q_1)(H - 2x)g\right] \pm a \sum m \;\; [N] \tag{13}$$

The functional based on which the electric energy consumption may be minimized during a race, may be established as follows:

$$\exists(x,a) = \int_0^T f(x,a)dt = \int_0^T F_i^2 dt \tag{14}$$

In order to simplify expression (4) of the peripheral force, the following transformations are made:

$$\begin{aligned} F &= k \cdot Q_u \cdot g + (q - q_1)g(H - 2x) + a\sum m \\ F &= A + D(H - 2x) + a\sum m = A + DH - 2Dx + a\sum m \end{aligned} \tag{15}$$

where $A = k \cdot Q_u \cdot g$; $D = (q - q_1)g$; only the positive sign has been considered for the acceleration.

By replacing the expression of the force it results:

$$\begin{aligned} f(x,a) = {} & (A + DH)^2 - 4D(A + DH)x + 4D^2x^2 \\ & + 2a(A + DH)\sum m - 4Dxa\sum m + a^2(\sum m)^2 \end{aligned} \tag{16}$$

Using the relation between the actual force and the quantity of heat developed within the reeling of the motor during a transportation cycle, the actual force expression (equivalent) may be used as an optimization criterion. Therefore, between functional (1) and the actual force there is the following relation:

$$F_{ef} = \sqrt{\frac{\int_0^T F^2 dt}{T_{ef}}} = \sqrt{\frac{\int_0^T f(x,a)dt}{T_{ef}}} \tag{17}$$

The beginning and the end of a transport cycle are characterized by the following conditions:

$$x(0) = 0; x(T) = H; v(0) = 0; x'(T) = v(T) = 0 \tag{18}$$

3. RESTRICTIONS ON THE TRANSPORT CYCLE

In optimizing the parameters of the transport cycle the respect of a series of technical prescriptions is imposed in order to ensure the continuous operation in full safety conditions. The variation of kinematics parameters (speed and acceleration) during a transport cycle is defined by the diagram of speed (tachogram) as well as by the diagram of the acceleration, characterized by the relations:

$$\begin{aligned}
&\int_0^T x'(t)dt = \int_0^T v(t)dt = H \leftrightarrow a)\\
&x'(0) = v(0) = x'(T) = v(T) = 0 \leftrightarrow b)\\
&x'(t) = v(t) \le v_{adm} \leftrightarrow c)\\
&x''(t) = \frac{dv(t)}{dt} = a \le a_{adm} \leftrightarrow d)\\
&x''(t) \le \rho_{adm} \leftrightarrow e)\\
&\frac{t_2}{T} \ge 0{,}6 \leftrightarrow f)
\end{aligned} \tag{19}$$

Conditions (19, a) and (19, b) define the requirements regarding movement and speed: at the end of the cycle, the space undergone by the transport enclosures has to be equal to the length of the transport race; the speed, both at the beginning of the movement as well as at the end of the race has to be null. Conditions (19, c) and (19, d) are defined by the technical prescriptions regarding the speed limit and acceleration with their maximum admissible values. Condition (19, e) limits the maximum value of the variation of the force within the time unit, seldom used measure during the automated control of vertical transport installations. Condition (19, f) is imposed by the cooling off of the electric motors through their own ventilation. The power of the actuating motor needs to satisfy the following criteria:

$$P_{ef} = \frac{F_{ef} \cdot v_{max}}{1000\ \eta_a} = \frac{v_{max}}{1000\ \eta_a}\sqrt{\frac{\int_0^T F^2 dt}{T_{ef}}} \le P_M \tag{20}$$

$$\frac{P_{max}}{P_{ef}} = \frac{F_{max}}{F_{ef}} \le \gamma \tag{21}$$

where: γ is the overload admissible coefficient (for asynchronous motors; for continuous current motors); F_{max} is the maximum value of the peripheral force appearing during the transport race; P_{max} is the power corresponding to the maximum force.

Two models based on relations (17) and functional (1) may be used for the optimisation:

• The optimisation model with the limit conditions (19, a) and (19, b);

• The optimisation model with all the kinematics restrictions imposed by the motor given by relations (17) and (18).

The first model covers criterion (11) and the limit conditions (16). A practical model needs therefore to consider all the restrictions, such as the second one foresees.

Therefore the amendment of functional (11) and the optimisation criterion (15) needs to be made, dividing the transport cycle in several according to the expression:

$$F_{ef} = \sqrt{\frac{\sum_{i=1}^{n}\int_{t_{ii}}^{t_{fi}} F^2 dt}{T_{ef}}} = \sqrt{\frac{\sum_{i=1}^{n}\int_{t_{ii}}^{t_{fi}} f(x,a)\,dt}{T_{ef}}} \quad [N] \tag{20}$$

where: n is the number of phases of the extraction cycle; t_{ii} is the beginning of all n phases; t_{fi} is the ending of all n phases.

4. THE EXTREMES OF THE OPTIMISATION FUNCTIONAL; EULER-POISSON EQUATIONS OF THE FUNCTIONAL

The establishment of the function characterizing the law of variation of space *x(t)*, considering that the integral (1) represents a superior order function related to the first derivative, may be made using the Euler-Poisson equation. The equation (23) adapted for the present case is:

$$\frac{\partial f}{\partial x} - \frac{d}{dt}\left(\frac{\partial f}{\partial x'}\right) + \frac{d^2}{dt^2}\left(\frac{\partial f}{\partial x''}\right) = 0 \tag{23}$$

Obtaining therefore:

$$\frac{d^4x}{dt^4} + \frac{4D}{\sum m}\frac{d^2x}{dt^2} + \frac{16D}{(\sum m)^2}x = \frac{2D(A-DH)}{(\sum m)^2} \tag{24}$$

or

$$\frac{d^4x}{dt^4} + \lambda\frac{d^2x}{dt^2} + \frac{\lambda^2}{4}x = \psi \tag{25}$$

where $\lambda = \frac{4D}{\sum M}$ and $\psi = \frac{2D(A-DH)}{(\sum m)^2}$;

Considering that the difference in weight between the transport cable and the balance one is characterized by *D*, three cases may be distinguished in solving the above presented equation

a) $D=g(q-q_1)>0$ - unbalanced installation; the roots of equation (25) are real;

b) $D=g(q-q_1)<0$ - dynamically balanced installation; the roots of equation (25) are imaginary;

c) $D=g(q-q_1)=0$ - statically balanced equation.

For D = 0, based on expressions (17) and (24) the following are obtained:

$$\frac{d^4x}{dt^4}=0 \tag{26}$$

Unbalanced installation (D > 0)

For this case, the solutions of equation (26), space, speed, acceleration and the third derivative of space in relation to time are the following:

$$\left.\begin{aligned}
&x=e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta;\\
&x'=v=C_1\alpha e^{\alpha t}+C_2e^{\alpha t}(1+\alpha t)-C_3\alpha e^{-\alpha t}+C_4e^{-\alpha t}(\alpha t-2);\\
&x''=a=C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2);\\
&x'''=\rho=\alpha^3e^{\alpha t}(C_1+C_2t)+3C_2\alpha^2e^{\alpha t}-\alpha^3e^{-\alpha t}(C_3+C_4t)+3C_4\alpha^2e^{-\alpha t}.
\end{aligned}\right\} \tag{27}$$

where: $\alpha=\sqrt{\left|-\frac{\lambda}{2}\right|};\beta=\frac{4\psi}{\lambda^2}$; C_i – integration constancies, i = 1; 2; 3; 4.

Statically balanced installation (D = 0)

For this case, the solutions of equation (26) are:

$$\left.\begin{aligned}
&x=C_1+C_2t+C_3t^2+C_4t^3 \quad x''=2C_3+6C_4t\\
&x'=C_2+2C_3t+3C_4t^2 \quad x'''=6C_4
\end{aligned}\right\} \tag{28}$$

where C_i are integration constancies, i = 1; 2; 3; 4.

Dynamically balanced equation (D < 0)

In this case, the solutions of equation (26) may have the following form:

$$\left.\begin{aligned}
&x=\cos\alpha t(C_1+C_2t)+\sin\alpha t(C_3+C_4t)+\beta;\\
&x'=v=\sin\alpha t(-C_1\alpha-C_2\alpha t+C_4)+\cos\alpha t(C_2-C_3\alpha+C_4\alpha t);\\
&x''=a=\cos\alpha t(-C_1\alpha^2-C_2\alpha^2t+2C_4\alpha)-\sin\alpha t(2C_2\alpha+C_3\alpha^2+C_4\alpha^2t);\\
&x'''=\alpha^3(C_1+C_2t)\sin\alpha t-\alpha^3(C_3+C_4t)\cos\alpha t-3C_2\alpha^2\cos\alpha t-3C_4\alpha^2\sin\alpha t.
\end{aligned}\right\} \tag{29}$$

where: $\alpha = \sqrt{\left|-\frac{\lambda}{2}\right|}$; C_i – integration constancies, i = 1; 2; 3; 4.

4.1. Optimum transport cycle with limit conditions

Mathematically speaking, the optimization of the transport cycle consists in founding the function *x(t)*, the law of movement, ensuring the minimum of the integral:

$$F_{ef} = \sqrt{\frac{\int_0^T f(x,a)dt}{T_{ef}}} = min$$

The case of unbalanced installations (D > 0)

Based on the solution of the equation given by expression (15) and the initial conditions, a four-equation system is formed in order to determine the integration constancies. Following the solution of this equation system, the integration constancies are:

$$\left.\begin{aligned} &C_1 = -C_3 - \beta \\ &C_2 = 2C_3\alpha - C_4 + \alpha\beta \\ &C_3 = \frac{a_3}{a_1} - \frac{a_2}{a_1}C_4 \\ &C_4 = \frac{a_1 b_3 - a_3 b_1}{a_1 b_2 - a_2 b_1} \end{aligned}\right\} \tag{30}$$

$$\left.\begin{aligned} &a_1 = e^{-\alpha T} - e^{\alpha T} + 2\alpha T e^{\alpha T} \\ &a_2 = T\,e^{-\alpha T} - T\,e^{\alpha T} \\ &a_3 = H - \beta + \beta e^{\alpha T} - \alpha\beta T\,e^{-\alpha T} \\ &b_1 = \alpha\,e^{-\alpha T} + 2\alpha^2 T e^{\alpha T} + \alpha\,e^{-\alpha T} \\ &b_2 = e^{-\alpha T} - \alpha T\,e^{-\alpha T} - e^{-\alpha T} - \alpha T\,e^{\alpha T} \\ &b_3 = -\alpha^2\,\beta T\,e^{\alpha T} \end{aligned}\right\} \tag{31}$$

The case of statically balanced installations (D = 0) Proceeding analogically, based on the solution of the equation given by expression (28), the integration constancies are:

$$C_1 = C_2 = 0;\ C_3 = \frac{3H}{T^2};\ C_4 = -\frac{2H}{T^3} \tag{32}$$

<u>The case of dynamically balanced installations (D < 0)</u> Considering the relations (17), the values of the integration constancies are:

$$\begin{aligned}&C_4=\frac{a_1b_3-a_3b_1}{a_1b_2-a_2b_1};\ C_3=\frac{a_3}{a_1}-\frac{a_2}{a_1}C_4\\&C_2=-C_3\alpha;\ C_1=\beta\end{aligned}\tag{33}$$

where:

$$\begin{aligned}&a_1=sin\alpha T-\alpha T\,cos\,\alpha T;\ a_2=T\,sin\alpha T\\&a_3=H-\beta+\beta cos\,\alpha\,t;\\&b_1=a^2T\,sin\alpha T+\alpha T\,cos\,\alpha T\\&b_2=sin\alpha T+\alpha T\,cos\,\alpha T;\ b_3=-\alpha\beta\,sin\alpha\,t\end{aligned}\tag{34}$$

4.2. The optimum transport cycle with all technological restrictions

Functional (10) will be adjusted with all the restrictions (6) imposed by the kinematics installation. Considering a three-period transport cycle, where the second period is characterized by constant speed, the limit conditions for each period may be explained as follows:

During the first period (the acceleration period)

$$\begin{aligned}&x(0)=0;\ \ x(t_1)=h_1;\ \ x'(0)=v(0)=0\\&x'(t_1)=v(t_1)=v_{max}\end{aligned}\tag{35}$$

During the second period (constant speed operation)

$$\begin{aligned}&x(t_1)=h_1;\ \ \ x(t_1+t_2)=h_1+h_2\\&x'(t_1)=v(t_1)=x'(t_1+t_2)=v(t_1+t_2)=v_{max}\end{aligned}\tag{36}$$

During the third period (the deceleration period)

$$\begin{aligned}&x(t_1+t_2)=h_1+h_2;\ \ x(t_1+t_2+t_3)=H\\&x'(t_1+t_2)=v(t_1+t_2)=v_{max}\\&x'(t_1+t_2+t_3)=v(t_1+t_2+t_3)=0\end{aligned}\tag{37}$$

where:

- t_i - represents the duration of the corresponding periods;

- h_i – is the distances undergone by the extraction containers during different periods.

In the same time, relations (35), (36) and (37) also need to comply with the following requirements:

$$t_1 + t_2 + t_3 = T;\ h_1 + h_2 + h_3 = H;\ \frac{t_2}{T} \geq 0{,}6$$

5. ADOPTED OPTIMIZATION MODEL

According to the expression (22), the following optimization model based on the equivalent force is adopted:

$$F_{ef} = \sqrt{\frac{\sum_{i=1}^{3}\int_{t_{ii}}^{t_{fi}} f(x,a)dt}{T_{ef}}} = min(!) \qquad (38)$$

The conditions from the start and the end of the cycle:

$$\begin{aligned} &x(t_{1,1} = 0) = 0; \quad x(t_{f3} = T) = H \\ &x'(0) = v(0) = 0; \quad x'(T) = v(T) = 0 \end{aligned} \qquad (39)$$

The requirements the actuating motor needs to comply with:

$$P_{ef} = \frac{v_{max}}{1000\eta_a}\sqrt{\frac{\sum_{i=1}^{3}\int_{t_{ii}}^{t_{fi}} f(x,a)dt}{T_{ef}}} \leq P_M;\ \frac{P_{max}}{P_{ef}} \leq \gamma \qquad (40)$$

Restrictions imposed on periods:

For the starting period:

$$\begin{aligned} &0 \leq t \leq t_1;\ 0 \leq x(t) \leq h_1;\ 0 \leq x'(t) \leq v_{max} \\ &a_{1min} \leq x''(t) \leq a_{1max} \\ &x''(t) \leq |\rho_{1max}|; \quad x(0) = 0; \quad x(t_1) = h_1 \\ &x'(0) = v(0) = 0; \quad x'(t_1) = v(t_1) = v_{max} \end{aligned} \qquad (41)$$

For the second period of constant speed operation:

$$\begin{aligned} &t_1 \leq t \leq t_1 + t_2; \quad h_1 \leq x(t) \leq h_1 + h_2 \\ &x'(t) = v_{max} = const.; \quad x''(t) = 0 \\ &x(t_1) = h_1; \quad x(t_1 + t_2) = h_1 + h_2 \end{aligned} \qquad (42)$$

For the deceleration period:

$$\begin{aligned}&t_1+t_2<t<T;\quad h_1+h_2\le x(t)\le H;\ \ 0\le x'(t)\le v_{max};\\&\left|a_{3min}\right|\le x''(t)=\left|a_{3max}\right|;\ \ x'''(t)=\left|\rho_{3max}\right|;\quad x(t_1+t_2)=h_1+h_2;\\&x(T)=H;\ \ x'(t_1+t_2)=v(t_1+t_2)=v_{max};\ \ x'(T)=v(T)=0\end{aligned}\tag{43}$$

Considering the expressions of x, x' and x'', the aspect of functional (16) for different balance degrees will be:

Unbalanced installation (D > 0)

$$\begin{aligned}f(x,a)=&(A+DH)^2-4D(A+DH)\cdot\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]+\\&+4D^2\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]^2+2(A+DH)\sum m\\&\cdot\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]-\\&-4D\sum m\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]\cdot\\&\cdot\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]+(\sum m)^2\cdot\\&\cdot\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]^2\end{aligned}\tag{44}$$

The C_i integration constants are determined using relations (30) and (31).

Statically balanced installation (D = 0)

$$f(x,a)=A^2+2A\sum m(2C_3+6C_4)+(2C_3+6C_4)^2(\sum m)^2\tag{45}$$

The C_i integration constants are determined using relation (32).

Dynamically balanced installation (D < 0)

$$\begin{aligned}f(x,a)=&(A+DH)^2-4D(A+DH)\left[(C_1+C_2t)cos\,\alpha t+(C_3+C_4t)sin\,\alpha t+\beta\right]+\\&+4D^2\left[(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]^2+2(A+DH)\sum m\\&\cdot\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]-\\&-4D\sum m\left[e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\right]\cdot\\&\cdot\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]+(\sum m)^2\cdot\\&\cdot\left[C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\right]^2\end{aligned}\tag{46}$$

The C_i integration constants are determined using relations (32) and (33).

Therefore, considering the three phases of the transport cycle, the numerator of expression (38) of the equivalent force may be written as follows:

$$\sum_{i=1}^{3}\int_{t_{ii}}^{t_{fi}} f(x,a)dt = \int_{0}^{t_1} f(x,a)dt + \int_{t_1}^{t_1+t_2} f(x,a)dt + \int_{t_1+t_2}^{T} f(x,a)dt \tag{47}$$

Considering the large volume of calculations, the digital integration of the components of expression (47) is imposed.

6. EXAMPLE

Based on the proposed method a C language software has been developed. Software which was tested for an extraction installation with cages with the following parameters:

- practical load extracted during a race: Q_u = 6000 kg;
- extraction depth: H = 480 m;
- the sum of reduced masses: Σm = 66368 kg;
- specific weight of the extraction cable: q = 5,77 kg/m;
- specific weight of the balance cable: q_1 = 6,72 kg/m;
- maximum acceleration at starting: a_{1max} = 0,8 m/s^2;
- maximum acceleration in breaking: a_{3max} = 1 m/s^2;
- maximum extraction speed: v_{max} = 9,35 m/s;
- operational period of extraction containers: T = 62 s;
- pause period between races: t_p = 20 s;
- transmission efficiency: η = 0,92.

In order to obtain a maximum efficiency, the following have been considered:

$$\frac{t_2}{T} = 0{,}6$$

and

$$t_1 = t_3 = 0{,}2 \cdot T$$

Eliminating q_1 for the unbalanced case and considering $q_1 = q$ for the statically balanced one, minimum values of the equivalent force and the actuating power have resulted with approximately 10% smaller than the classic method.

Figure 1 presents a print screen of the results obtained.

7. CONCLUSIONS

Analyzing the optimization trials of electric operation of extraction installations, presented in the specialty literature, it is observed that these are valid only for trapezoid tachograms (with constant accelerations and linear variation of speed in extreme periods). There is no certainty that this type of variation is optimum for ensuring the value of the minimum power. Imposing from the beginning a trapezoid form of the tachogram does not have any scientific justification, being made empirically;

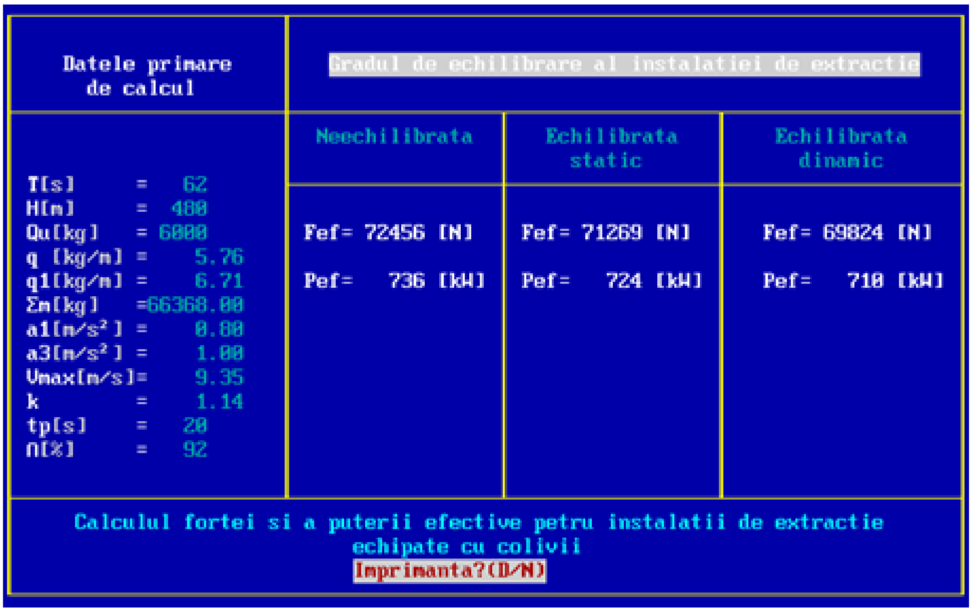

Fig.1. Print screen of obtained results for an extraction installation with cages

In order to minimize the actuating power of the extraction installations, the method of the calculus of variations is used, establishing an adequate mathematical model;

In order to use the proposed optimization method, the definition of the optimization and restriction functional was imposed. The optimization functional is based on the peripheral force of the cable actuating organism results from the general equation of dynamics;

The solutions of Euler-Poisson equations of the optimization functional differ depending the degree of balance of the installation;

The important determination volume for integrating the optimization functional implies the use of computers. The software developed in C language and also experimented proved itself to be a fast tool for practical calculations;

The proposed method is an operative and precise one and may serve to verify and design the extraction installations, determining the optimum functional parameters.

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