

COMPARISON THE RESULTS BASED ON INTERVAL NUMBERS, FUZZY SET METHOD AND PROBABILITY APPROACH

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Abstract: The contribution deals with simulation of input system variables by using the three approaches – interval numbers, fuzzy set based methods and probabilistic methods. The presented examples (computation of the natural frequencies of mechanical system and position and acceleration of crank mechanism) are using by this approaches (by means of INTLAB and Monte Carlo method). The results are compared. Attention is done to the potential miss-interpretation of the results, in case interpretation of interval numbers as a realization of uniform distributed random variable, or in case interpretation triangular fuzzy number as a realization random variable with triangular probability density function.

Keywords: interval number, fuzzy set, random variable, INTLAB, Monte Carlo Method

1. INTRODUCTION

Mechanical equipment modeling uses the variables, which describes loading, geometrical and material properties. These variables take either an only one value or a lot of values. We will assume that these variables are expressed in second way. We intend to compare the results of the three selected approaches to modeling and these variables will express by the interval numbers [1,2], fuzzy set methods [3-5] and

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random variables with probability density functions.

The choice of simulation techniques is established on the basis of our knowledge about object. Simply said - if we know only the range of variables then we use technology of interval number else if we are able to determine membership function which is define a variable relationship to the range then we use the fuzzy set based methods or if we know probability density function of the variable in the range then we use probability approach. There is no need to reproach someone – it is a good decision. We wish to remind you that the problem of results interpretation may ensue it.

Two examples are presented: dynamic model and position and acceleration of crank mechanism. The interval numbers and fuzzy set approach are solved by Matlab-Intlab. Using Monte Carlo method is solved probability approach.

2. INTERVAL ARITHMETIC AND INTLAB

The idea of bounding rounding errors using intervals was introduced by several people in the 1950's. Interval algorithms may be used in most areas of numerical analysis, and are used in many applications such as engineering problems and computer-aided design.

This part presents fundamental principles of interval arithmetic and its applications to MATLABs tool – INTLAB [1, 6-8].

Intervals will be represented by boldface, with the brackets „[.]“ used for intervals defined by an upper bound and a lower bound. Underscores will be used to denote lower bounds of intervals and overscores will denote upper bounds. For intervals defined by a midpoint and a radius the brackets „<.>“ will be used.

A real interval $\mathbf{x}$ is a nonempty set of real numbers

$$\mathbf{x} = [\underline{x}, \overline{x}] = \{x \in \mathbf{R} : \underline{x} \leq x \leq \overline{x}\}, \quad (1)$$

where $\underline{x}$ is called the *infimum* and $\overline{x}$ is called the *supremum*. The set of all intervals over $\mathbf{R}$ is denoted by $\mathbf{IR}$ where

$$\mathbf{IR} = \{[\underline{x}, \overline{x}] : \underline{x}, \overline{x} \in \mathbf{R}, \underline{x} \leq \overline{x}\}. \quad (2)$$

The *midpoint* of $\mathbf{x}$,

$$mid(\mathbf{x}) = \bar{x} = \frac{1}{2}(\underline{x} + \overline{x}), \quad (3)$$

the *radius* of $\mathbf{x}$,

$$rad(\mathbf{x}) = \frac{1}{2}(\overline{x} - \underline{x}), \quad (4)$$

may also be used to define an interval $\mathbf{x} \in \mathbf{IR}$. An interval with midpoint a and radius r will be denoted by $\langle a, r \rangle$.

The *absolute value* or the *magnitude* of an interval $\mathbf{x}$ is defined as

$$|\mathbf{x}| = \text{mag}(\mathbf{x}) = \max\{|\tilde{x}| : \tilde{x} \in \mathbf{x}\}, \quad (5)$$

and the *magnitude* of $\mathbf{x}$ is defined as

$$\text{mig}(\mathbf{x}) = \min\{|\tilde{x}| : \tilde{x} \in \mathbf{x}\}. \quad (6)$$

These can both be calculated using the end points of $\mathbf{x}$ by

$$\text{mag}(\mathbf{x}) = \max\{|\underline{x}|, |\bar{x}|\}, \quad (7)$$

$$\text{mig}(\mathbf{x}) = \begin{cases} \min\{|\underline{x}|, |\bar{x}|\} & \text{if } \mathbf{x} \neq 0 \\ 0 & \text{otherwise} \end{cases}. \quad (8)$$

An interval $\mathbf{x}$ is a *subset* of an interval $\mathbf{y}$, denoted by $\mathbf{x} \subseteq \mathbf{y}$, if and only if $\underline{y} \leq \underline{x}$ and $\bar{y} \geq \bar{x}$. The relation $\mathbf{x} < \mathbf{y}$ means that $\bar{x} < \underline{y}$, and other inequalities are defined in a similar way.

Interval arithmetic operations are defined on $\mathbf{IR}$ such that the interval result encloses all possible real results. Given $\mathbf{x} = [\underline{x}, \bar{x}]$ and $\mathbf{y} = [\underline{y}, \bar{y}]$, the four elementary operations are defined by

$$\begin{aligned} \mathbf{x} + \mathbf{y} &= [\underline{x} + \underline{y}, \bar{x} + \bar{y}] \\ \mathbf{x} - \mathbf{y} &= [\underline{x} - \bar{y}, \bar{x} - \underline{y}] \\ \mathbf{x} \times \mathbf{y} &= [\min\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}, \max\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}] \\ \mathbf{x} \div \mathbf{y} &= \mathbf{x} \times 1/\mathbf{y}, \\ 1/\mathbf{x} &= [1/\bar{x}, 1/\underline{x}] \quad \text{if } \underline{x} > 0 \text{ or } \bar{x} < 0. \end{aligned} \quad (9)$$

For the elementary interval operations, division by an interval which containing zero is not defined. It is often useful to remove this restriction to give what is called *extended interval arithmetic*. Extended interval arithmetic leads to the following rules. If $\mathbf{x} = [\underline{x}, \bar{x}]$ and $\mathbf{y} = [\underline{y}, \bar{y}]$ with $\underline{y} \leq 0 \leq \bar{y}$ and $\underline{y} < \bar{y}$, then the rules for division are as follows

$$\mathbf{x/y} = \left\{ \begin{array}{ll} \left[\frac{\bar{x}}{\bar{y}}, \infty \right] & \text{if } \bar{x} \leq 0 \text{ and } \bar{y} = 0 \\ \left[-\infty, \frac{\bar{x}}{\bar{y}} \right] \cup \left[\frac{\bar{x}}{\bar{y}}, \infty \right] & \text{if } \bar{x} \leq 0 \text{ and } \bar{y} < 0 < \bar{y} \\ \left[-\infty, \frac{\bar{x}}{\bar{y}} \right] & \text{if } \bar{x} \leq 0 \text{ and } \bar{y} = 0 \\ \left[-\infty, \infty \right] & \text{if } \bar{x} < 0 \bar{x} \\ \left[-\infty, \frac{\bar{x}}{\bar{y}} \right] & \text{if } \bar{x} \geq 0 \text{ and } \bar{y} = 0 \\ \left[-\infty, \frac{\bar{x}}{\bar{y}} \right] \cup \left[\frac{\bar{x}}{\bar{y}}, \infty \right] & \text{if } \bar{x} \geq 0 \text{ and } \bar{y} < 0 < \bar{y} \\ \left[\frac{\bar{x}}{\bar{y}}, \infty \right] & \text{if } \bar{x} \geq 0 \text{ and } \bar{y} = 0 \end{array} \right\} \quad (10)$$

For further rules for extended interval arithmetic see [1, 6, 7].

3. FUZZY SET APPROACH

By Zadeh [4] was formulated the initial theory of fuzzy sets. A fuzzy set x is the set with boundaries that are not sharply defined. A function, called membership function (MSF), signifies the degree to which each member of a domain X belongs to the fuzzy set x []. For a fuzzy variable $x \in \langle x1, x2 \rangle$, (or $x \in x$), the membership function is defined as $\mu(x)$. If $\mu(x)=1$, x is definitely a member of the x [2, 5, 9]. If $\mu(x)=0$, x is definitely not a member of the x . For every x with $0 < \mu(x) < 1$, the membership is not certain. Typical membership functions of fuzzy sets are shown on figures 1 and 2.

$$\mu_A(x) = \begin{cases} 0, & x \notin \langle a_1, a_3 \rangle \\ \frac{x - a_1}{a_2 - a_1}, & x \in \langle a_1, a_2 \rangle \\ 1, & x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & x \in \langle a_2, a_3 \rangle \end{cases} \quad (11)$$

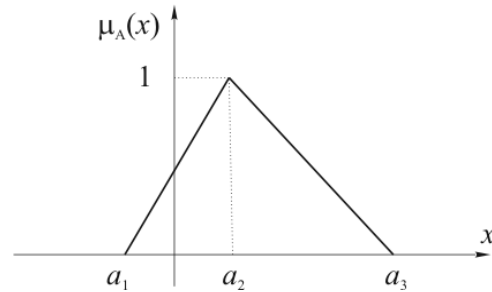


Fig. 1. MSF for a triangular fuzzy number.

$$\mu_A(x) = \begin{cases} 0, & x \notin \langle a_1, a_4 \rangle \\ \frac{x - a_1}{a_2 - a_1}, & x \in \langle a_1, a_2 \rangle \\ 1, & x \in \langle a_2, a_3 \rangle \\ \frac{a_4 - x}{a_4 - a_3}, & x \in \langle a_3, a_4 \rangle \end{cases} \quad (12)$$

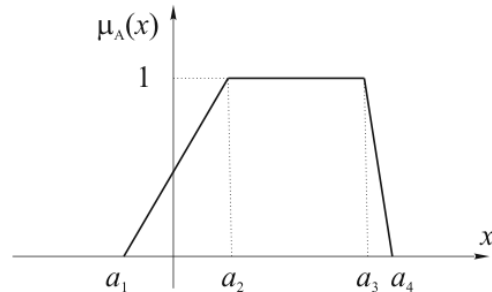


Fig. 2. MSF for a trapezoidal fuzzy number.

By fuzzy technique, the complete information about the uncertainties in the model can be included and one can demonstrate how these uncertainties are processed through the calculation procedure in MATLAB [1, 7].

4. PROBABILISTIC APPROACH

Probabilistic approach is based on the transformation of random variables.

Lets $\mathbf{X} = (X_1, \dots, X_n)^T$ be a n -dimensional random variable with joint probability distribution function $f_{\mathbf{X}}(x_1, \dots, x_n)$. We find joint probability distribution function $g_{\mathbf{Y}}(y_1, \dots, y_n)$ of n -dimensional random variable $\mathbf{Y} = (Y_1, \dots, Y_n)^T$, where $Y_j = h_j(X_1, \dots, X_n)$, $j = 1, \dots, n$, and $\mathbf{h} = (h_1, \dots, h_n)^T$ is 1:1 transformation of $\mathbf{X}$ into $\mathbf{Y}$.

Lets inverse function $\mathbf{h}^{-1}: \mathbf{x} = \mathbf{h}^{-1}(\mathbf{y})$ is differentiable and Jacobian of the transformation J

$$J = \begin{vmatrix} \frac{\partial h_1^{-1}}{\partial y_1} & \dots & \frac{\partial h_1^{-1}}{\partial y_n} \\ \dots & \dots & \dots \\ \frac{\partial h_n^{-1}}{\partial y_1} & \dots & \frac{\partial h_n^{-1}}{\partial y_n} \end{vmatrix}, \quad J \neq 0, \quad \frac{1}{J} = \begin{vmatrix} \frac{\partial h_1}{\partial x_1} & \dots & \frac{\partial h_1}{\partial x_n} \\ \dots & \dots & \dots \\ \frac{\partial h_n}{\partial x_1} & \dots & \frac{\partial h_n}{\partial x_n} \end{vmatrix}. \quad (13)$$

The joint probability distribution function $g_{\mathbf{Y}}$

$$g_{\mathbf{Y}}(\mathbf{y}) = g_{\mathbf{Y}}(y_1, \dots, y_n) = f_{\mathbf{X}}[h_1^{-1}(y_1, \dots, y_n), \dots, h_n^{-1}(y_1, \dots, y_n)]J = f_{\mathbf{X}}[\mathbf{h}^{-1}(\mathbf{y})]J \quad (14)$$

Calculation of the function $g_{\mathbf{Y}}$ can be very difficult. We will use Monte Carlo method to the estimation of parameters of random variable $\mathbf{Y}$.

Probability density function $f_X(x)$ is for uniform distribution of random variable X on the interval (a_1, a_3) , $X \sim \text{Unif}(a_1, a_3)$, Fig.3:

$$f_X(x) = \begin{cases} 0, & x \notin \langle a_1, a_3 \rangle \\ \frac{1}{a_3 - a_1}, & x \in \langle a_1, a_3 \rangle \end{cases} \quad (15)$$

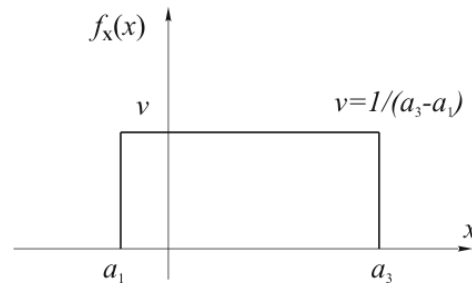
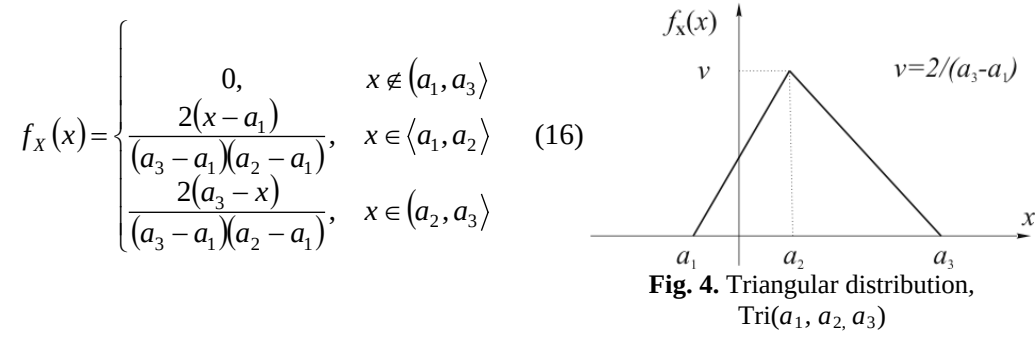


Fig. 3. Uniform distribution $\text{Unif}(a_1, a_3)$

Probability density function $f_X(x)$ is for triangular distribution of random variable X on the interval (a_1, a_3) , $X \sim \text{Tri}(a_1, a_2, a_3)$, Fig.4:



5. DYNAMIC MODEL

Vehicle dynamic models are often characterized by uncertain system parameters. Main goal of this example will be to explain difference between three different approaches to the analysis the influence of the uncertain parameters on the natural frequencies case of the model.

Let's consider the 7-DOFs model of the agricultural tractor (Fig. 5).

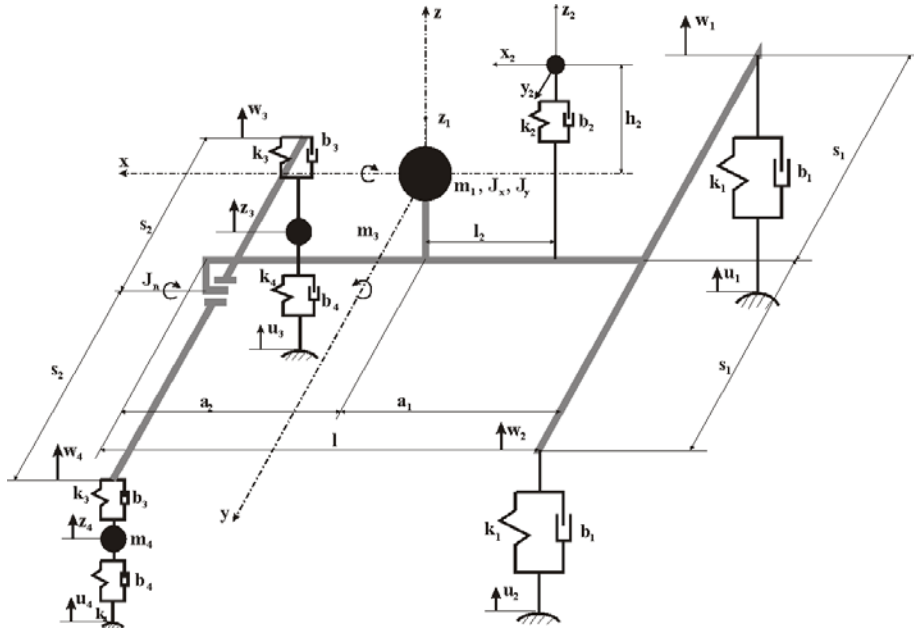


Fig.5. The dynamic model of agricultural tractor

The equation of motion is

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{B} \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{p}, \quad (17)$$

and $\mathbf{M}$ is diagonal matrix of mass, $\mathbf{B}$ is symmetric matrix of damping (we will consider model without damping, $\mathbf{B} = 0$), $\mathbf{K}$ is symmetric stiffness matrix and $\mathbf{p}$ is vector of excitation. In modal analysis of non-conservative and asymmetric systems the eigenvalues are complex values [10].

We will consider only conservative, non-damping system and problem of natural frequencies can be described by equation

$$(\mathbf{K} - \lambda \cdot \mathbf{M}) \cdot \mathbf{q}_0 = \mathbf{0}, \quad (18)$$

and

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_4 \end{bmatrix}, \quad (19)$$

$$\mathbf{K} = \begin{bmatrix} 2k_1 + k_2 + 2k_3 & 0 & 2a_1k_1 + l_2k_2 - 2a_2k_3 & 0 & -k_2 & -k_3 & -k_3 \\ 0 & 2s_1^2k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2a_1^2k_1 + l_2^2k_2 - 2a_2^2k_3 & 0 & -l_2k_2 & a_2k_3 & a_2k_3 \\ 0 & 0 & 0 & 2s_2^2k_3 & 0 & s_2k_3 & -s_2k_3 \\ 0 & 0 & 0 & 0 & k_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_3 + k_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_3 + k_4 \end{bmatrix}, \quad (20)$$

$\mathbf{q}_0$ is eigenvector and λ is eigenvalue.

Lets all inputs parameters without stiffness is constants. The stiffness [Nm-1] are simulated:

1. Deterministic - constant values:

$$k_1 = 3 \cdot 10^5, k_2 = 2 \cdot 10^4, k_3 = 1 \cdot 10^5, k_4 = 2 \cdot 10^5,$$

2. Interval numbers:

$$k_1 = \langle 2,9399 \cdot 10^5, 3,0601 \cdot 10^5 \rangle, k_2 = \langle 1,9599 \cdot 10^4, 2,0401 \cdot 10^4 \rangle,$$

$$k_3 = \langle 0,9799 \cdot 10^5, 1,0201 \cdot 10^5 \rangle, k_4 = \langle 1,9599 \cdot 10^5, 2,0401 \cdot 10^5 \rangle,$$

3. Triangular fuzzy numbers:

$$k_1 = \langle 2,9399.10^5, 3.10^5, 3,0601.10^5 \rangle, \quad k_2 = \langle 1,9599.10^4, 2.10^4, 2,0401.10^4 \rangle,$$

$$k_3 = \langle 0,9799.10^5, 1.10^5, 1,0201.10^5 \rangle, \quad k_4 = \langle 1,9599.10^5, 2.10^5, 2,0401.10^5 \rangle,$$

4. Random variables:

4.1. Uniform distribution Unif(a,b) in the interval (a,b):

$$k_1 \sim \text{Unif}(2,9399.10^5, 3,0601.10^5), \quad k_2 \sim \text{Unif}(1,9599.10^4, 2,0401.10^4),$$

$$k_3 \sim \text{Unif}(0,9799.10^5, 1,0201.10^5), \quad k_4 \sim \text{Unif}(1,9599.10^5, 2,0401.10^5),$$

4.2. Triangular distribution Tri(a,c,b):

$$k_1 \sim \text{Tri}(2,9399.10^5, 3.10^5, 3,0601.10^5), \quad k_2 \sim \text{Tri}(1,9599.10^4, 2.10^4, 2,0401.10^4)$$

$$k_3 \sim \text{Tri}(0,9799.10^5, 1.10^5, 1,0201.10^5), \quad k_4 \sim \text{Tri}(1,9599.10^5, 2.10^5, 2,0401.10^5)$$

A difference between triangular fuzzy number and triangular distribution can be described for k_1 :

a) If $k_1 = \langle 2,9399.10^5, 3.10^5, 3,0601.10^5 \rangle$, then the triangular height (for graphic interpretation of fuzzy number) in the point 3.10^5 equal sing 1,

b) If $k_1 \sim \text{Tri}(2,9399.10^5, 3.10^5, 3,0601.10^5)$, then the triangular height (for graphic interpretation of probability density function) in the point 3.10^5 equal sing $1/601$. (The area under triangular have to be 1.)

The first five natural frequencies – eigenfrequencies are presented in the table 1. (For models 1. – 3.), Result is presented in the table 2 for model number 4.2. The first three eigenfrequencies for triangular density functions and fuzzy approach are depicted on the figure 6a-c. The first three eigenfrequencies for uniform density functions are depicted on the figure 7a-c. A histogram is normalized by maximal value in container.

Tab. 1. The first five eigenfrequencies are presented for constant, interval and fuzzy model

	Constant model	Interval model	Fuzzy model
f_1	1.8606	<1.8419; 1.8791>	<1.8419, 1.8606, 1.8792>
f_2	2.1126	<2.0678 2.1565>	<2.0914, 2.1126, 2.1337>
f_3	2.2508	<2.2282 2.2732>	<2.2281, 2.2508, 2.2732>
f_4	2.5165	<2.4912 2.5415>	<2.4911, 2.5165, 2.5416>
f_5	7.5494	<7.4735 7.6245>	<7.4735, 7.5494, 7.6249>

Tab. 2. The first five eigenfrequencies are presented for triangular probability density function

Natural Frequency	Mean value	Confidence interval		
		99%	95%	90%
1.	1.86057	(1.8486, 1.8726)	(1.8513, 1.8699)	(1.8527, 1.8684)
2.	2.11261	(2.1000, 2.1252)	(2.1029, 2.1223)	(2.1044, 2.1208)
3.	2.25076	(2.2320, 2.2694)	(2.2362, 2.2652)	(2.2385, 2.2631)
4.	2.51641	(2.4956, 2.5373)	(2.5002, 2.5326)	(2.5027, 2.5300)
5.	7.54933	(7.4862, 7.6127)	(7.5003, 7.5981)	(7.5081, 7.5903)

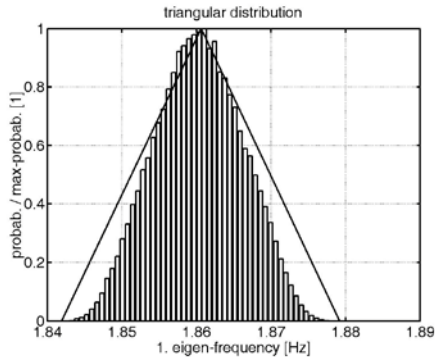


Fig. 6a. Fuzzy and triangular distribution.

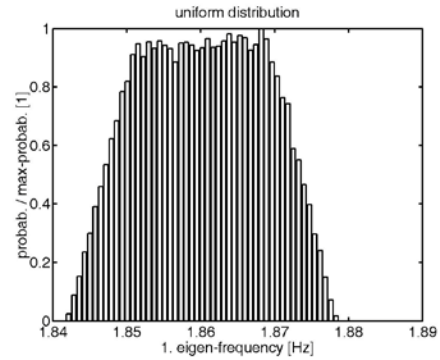


Fig. 7a. Uniform distribution.

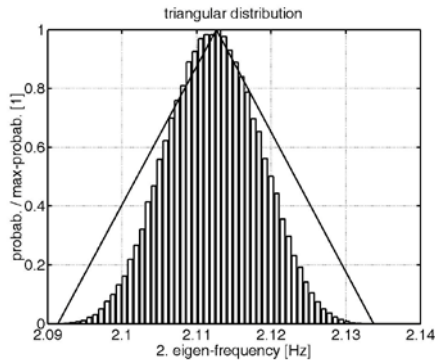


Fig. 6b. Fuzzy and triangular distribution.

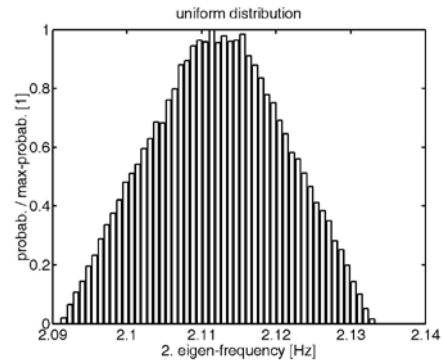


Fig. 7b. Uniform distribution.

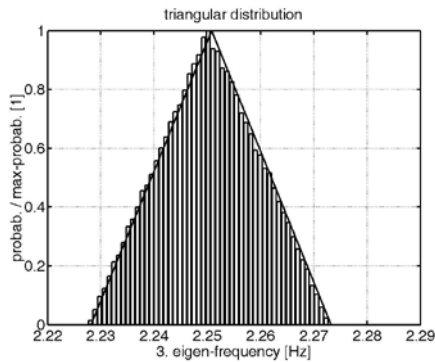


Fig. 6c. Fuzzy and triangular distribution.

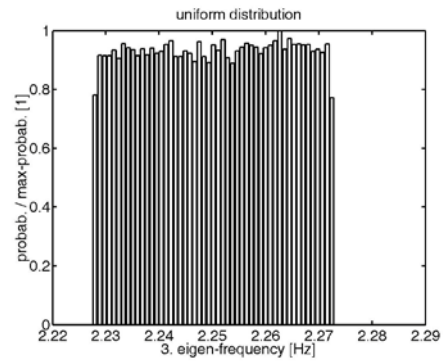


Fig. 7c. Uniform distribution.

A soft histogram shift in left is caused by option of containers. “The teeth” can be decreasing by increasing number of samples (105 samples). The probability density function is approximately triangular/rectangle for triangular/uniform probability

density function of input variables. Figures for 4th and 5th natural frequency are not depicted in this paper.

On the figure number 6a, 6b is obvious difference between fuzzy and probability approach results. There is visible difference in the range of interval for interpretations fuzzy versus probability too.

If we will obtain the results for interval approach and will extend interpretation of results in probability sense (interval number $\approx$ random variable with uniform distribution) then result have no uniform distribution. (Fig.7a, 7b)

6. CRANK MECHANISM

Let us consider crank mechanism [11] what is depicted in Fig. 8.

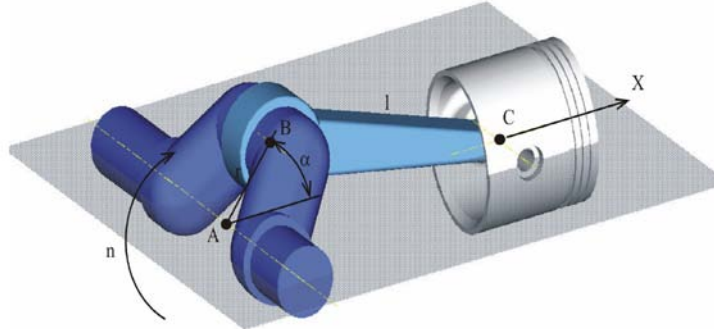


Fig. 8. Crank mechanism.

Let us variables are crank length $r = \overline{AB}$ and connecting rod length $l = \overline{BC}$. Piston position $u = \overline{AC}$ (origin of coordinate system is in point A) and piston acceleration a are determined, for different angle α and constant crank speed n .

The crank length and connecting rod length r, l [m] are simulated by:

1. Triangular fuzzy numbers:
 $r = \langle 0,018, 0,02, 0,022 \rangle, l = \langle 0,126; 0,14; 0,154 \rangle$

2. Random variables:
 Uniform distribution Unif(a,b) in the interval (a,b):
 $r \sim \text{Unif}(0,018 ; 0,022), l \sim \text{Unif}(0,126 ; 0,152),$

Triangular distribution Tri(a,c,b):
 $r \sim \text{Tri}(0,018 ; 0,02; 0,022), l \sim \text{Tri}(0,126 ; 0,14; 0,154),$

Piston positions are depicted in Fig. 9a-c, where Fig.9a are results for fuzzy approach, Fig.9b - triangular distribution and Fig.9c – uniform distribution.

Piston accelerations are depicted in Fig.10a-c, where Fig.10a are results for fuzzy approach, Fig.10b - triangular distribution and Fig.10c – uniform distribution.

The simulation is done for different value of angle α . For every value of angle are generated 10^6 samples. On the z-axis are depicted fuzzy numbers or normalized

probability density function. The peaks are in the near of inflection points and increasing of number samples is do not ensure decreasing these peaks. (The area under probability distribution function is equal one – for angle α .)

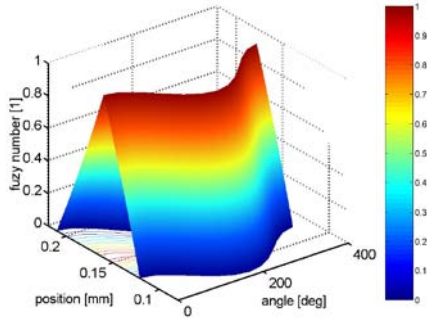


Fig. 9a. Position – fuzzy

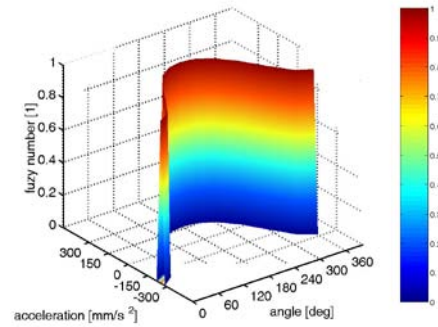


Fig. 10a. Acceleration – fuzzy

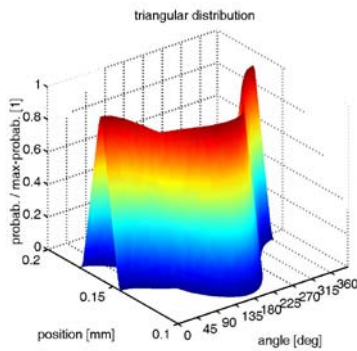


Fig. 9b. Position – triangular distribution.

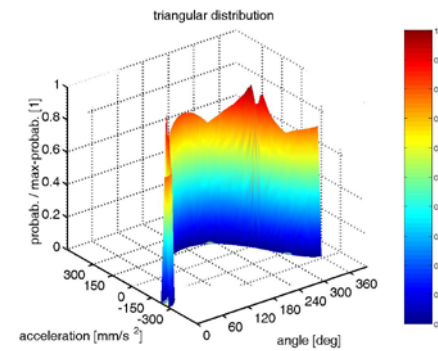


Fig. 10b. Acceleration – triangular distribution.

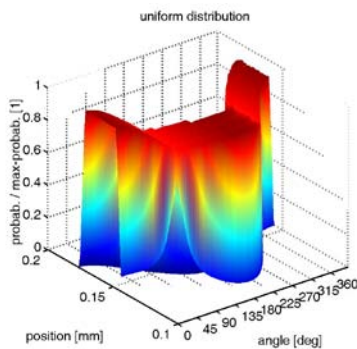


Fig. 9c. Position – uniform distribution.

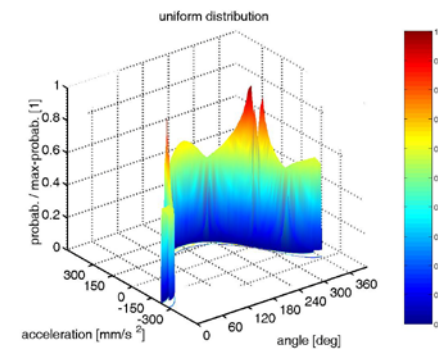


Fig. 10c. Acceleration – triangular distribution.

7. CONCLUSION

The paper presents comparison study of interval numbers, fuzzy set and probabilistic approaches from point of view results interpretation. Every of these accesses have itself advantage. Study is warned against risk of miss-interpretation of results, in the case interpretation of interval numbers as a realization of uniform distributed random variable, or in the case interpretation triangular fuzzy number as a realization random variable with triangular probability density function.

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