ELECTRO-DYNAMIC FORCES IN SHORT CIRCUIT
REGIME USING MATLAB - SIMULINK SOFTWARE

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Abstract: Nowadays, computer simulation is more and more used in engineering applications. Powerful IT software helps researchers from all engineering branches to gain fast and accurate results. One of these tools is represented by the MATLAB software package which is a powerful tool used worldwide in electrical engineering. The key features of MATLAB are matrix-oriented programming, excellent plotting capability and the included graphical environment – Simulink – which highly simplifies the control scheme design. This paper focuses on the MATLAB-Simulink software application for studying electro-dynamic forces. The single-phase short circuit and the three-phase short circuit are presented and for these situations the graphical correlation between these forces and time are presented for transitory phenomena. Simulation models and theoretical basement are presented also.

Keywords: electro-dynamic force, model, simulation, single-phase short circuit, three-phase short-circuit.

1. SINGLE-PHASE SHORT CIRCUITS

The electro-dynamic forces result in the distribution and transport of electric energy with two conductors and with three conductors when the short circuit occurs between three conductors [3], [11]. The mathematical expression of electro-dynamic forces for the transitory regime is:

\[ F = C l^2 (e^{-\lambda t} - \cos \omega t)^2 \] (1)

Where \( \lambda \) is the equivalent time constant, and:

\[ C = \frac{H_0 \cdot l}{2\pi} \cdot \phi_{cd} \cdot \phi(\frac{a}{l}) \] (2)

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In equation (2) $\varphi_{CD}$ is a function which depends on Dwight’s diagrams, and:

$$\varphi\left(\frac{a}{l}\right) = \sqrt{l + \frac{a^2}{l^2} - \frac{a}{l}}$$

(3)

where “l” is the length of the conductors and “a” is the distance between them.

Circuit Simulation Model: The explicit function (1) leads to the simulation model from (Fig.1), which generated the explicit diagram for dependence between electro-dynamic force and time (Fig.2) [8], [10]. The explicit diagrams have been obtained for specific values of the circuit parameter as follows:

$$R = 100\Omega; \ L = 2H; \ l = 100m; \ a = 0.5m; \ \varphi_{CD} = 0.8; \ I = 100A.$$

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Fig.1. Simulink model for single-phase short circuit

The subsystem 1 is shown in Figure 8.

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Fig.2. Force diagram for single-phase short circuit
2. THREE-PHASE SHORT CIRCUITS

The electro-dynamic forces occur in the transport and distribution of energy in three-phase systems. Short circuit is performed between the three active conductors of the system [2]. In Fig. 3 we have the case of three parallel conductors, the location and sign convention for the forces exerted on the median conductor, respectively on a side conductor [9].

![Fig.3. Disposition of conductors in the same plane: a) Forces exerted on the median conductors b) Forces exerted on the side conductor](image)

Force Exerted on the Median Conductor: If a short circuit between conductors is performed in three phase system, the electro-dynamic force acting on the median conductor is [5]:

\[
F_m = C \cdot i_1 \cdot (i_2 - i_3)
\]  

(4)

with the expressions of currents in the three conductors:

\[
\begin{align*}
  i_1 &= \hat{I} \cdot \left[ e^{-\lambda t} \cdot \sin \alpha + \sin(\omega t - \alpha) \right] \\
  i_2 &= \hat{I} \cdot \left[ e^{-\lambda t} \cdot \sin \left( \alpha + \frac{2\pi}{3} \right) + \sin \left( \omega t - \alpha - \frac{2\pi}{3} \right) \right] \\
  i_3 &= \hat{I} \cdot \left[ e^{-\lambda t} \cdot \sin \left( \alpha - \frac{2\pi}{3} \right) + \sin \left( \omega t - \alpha + \frac{2\pi}{3} \right) \right]
\end{align*}
\]

(5)

The force exerted on the median conductor is obtained by inserting the expressions of currents presented in equation (5) into equation (4):

\[
F_m = C \cdot \hat{I}^2 \cdot \left[ e^{-\lambda t} \cdot \sin \alpha + \sin(\omega t - \alpha) \right] \left[ e^{-\lambda t} \cdot \sin \left( \alpha + \frac{2\pi}{3} \right) + sin \left( \omega t - \alpha - \frac{2\pi}{3} \right) \right] + \\
\left[ e^{-\lambda t} \cdot \sin \left( \alpha - \frac{2\pi}{3} \right) + \sin \left( \omega t - \alpha + \frac{2\pi}{3} \right) \right]
\]


\[ F_m = \sqrt{3} \cdot C \cdot I^2 \cdot \left[ e^{-\lambda t} \cdot \sin(\omega t) + e^{-\lambda t} \cdot \sin(\omega t - \alpha) \right] \cdot \left[ e^{-\lambda t} \cdot \cos(\omega t - \alpha) - e^{-\lambda t} \cdot \cos(\omega t) - \frac{\sqrt{3}}{2} \cdot \cos(\omega t - \alpha) \right] \]

The maximum force occurs for the angle \( \alpha = \frac{\pi}{4} \). The final force expression is:

\[ F_m = \sqrt{3} \cdot C \cdot I^2 \cdot \left[ e^{-\lambda t} \cdot \sin(\omega t - \alpha) \right] \cdot \left[ e^{-\lambda t} - \sin(\omega t - \cos(\omega t - \alpha)) \right] \]

The simulation model of the force is presented in Fig. 4 and the generated diagram in Fig.5.

Fig.4. Simulink model for three-phase short circuit, median conductor
Subsystem 2 is shown in Fig. 9.

![Graph showing force diagram for three-phase short circuit, median conductor](image)

**Fig.5.** Force diagram for three-phase short circuit, median conductor

Force Exerted on the Side Conductor: If a short circuit between occurs between the conductors of a three phase system, the electro-dynamic force acting on the side conductor is [6]:

\[
F_i = C \cdot i_i \cdot \left( i_2 + \frac{i_3}{2} \right)
\]

(7)

If the previous calculation method is applied, this force becomes:

\[
F_i = -\frac{\sqrt{3}}{4} \cdot C \cdot I^2 \cdot \left[ e^{-\gamma_2} \cdot \sin \alpha + \sin(\omega t - \alpha) \right] \\
\cdot \left[ e^{-\gamma_2} \cdot \left( \sqrt{3} \cdot \sin \alpha - \cos \alpha \right) + \sqrt{3} \cdot \sin(\omega t - \alpha) + \cos(\omega t - \alpha) \right]
\]

(8)

The simulation model of the force is presented in Fig. 6 and the generated diagram is presented in Fig.7 [7].

![Simulink model for three-phase short circuit, side conductor](image)

**Fig.6.** Simulink model for three-phase short circuit, side conductor
Subsystem 3 is shown in Figure 10.

![Graph showing force diagram for three-phase short circuit, side conductor](image)

**Fig.7.** Force diagram for three-phase short circuit, side conductor

Simulation models have been designed so that input quantities are grouped separately to be modified and thus allow study of the influence of each input parameter on electro-dynamic forces for each situation [1], [4]. Each simulation model includes one subsystem. The three subsystems are presented below:

![Diagram showing subsystem 1](image)

**Fig.8.** Subsystem 1
3. CONCLUSIONS

In single-phase short circuits, the highest mechanical strain for devices occurs at the beginning of the transient regime, when the shock occurs (highest a-periodic component) after which the current gradually decreases. From the simulations there may
be observed that pulsations of force are uneven during the transient short-circuit and become equal after a sufficiently long period of time.

In case of three-phase short circuits, the forces on the median conductor are pulsed, in both directions. It is noticed that unlike the case of single-phase short circuit, for this one, the senses of force don’t remain the same, but change over time. The amplitude of the force reaches the highest value at the beginning of the transient regime, after which it stabilizes until it reaches the steady state value.

For three-phase short circuits, the amplitude of the force in this system is much higher than the steady state value, but the dominant action is the same: pushing outward the lateral conductors system.

**REFERENCES**


