PREMIUMS CALCULATION FOR LIFE INSURANCE

ANA PREDA, MARIANA GÎRBACI *

ABSTRACT: *The paper presents the techniques and the formulas used on international practice for establishing the premiums for a life policy. The formulas are generally based on a series of indicators named mortality indicators which mainly point out the insured survival probability, the death probability and life expectancy at certain age. I determined, using a case study, the unique net premium, the annual net premium for a survival insurance, whole life insurance and mixed life insurance.*

KEY WORDS: *premium; life insurance; mortality tables; commutation numbers.*

JEL CLASSIFICATION: *G22; G32; G35.*

1. INTRODUCTION

Actuarial science appeared from the eighteenth century through a combination of the interest rate with mortality tables, actuarial term appears once with the society "The Equitable" in 1762, and the first who founded the actuarial profession was William Morgan, who held this post at this society (Anghelache, 2006). The actuary has the most important role in calculating insurance premiums, "called so life insurance mathematician" (Ciurel, 2000) with the following major tasks: determining the insurance premium, determining mathematical reserves, the surrender values of capitalization products, new products development, and others. The main elements underlying the calculation of insurance premiums are mortality tables, general health of the population, age and sex of the insured, contract period, the level of sums insured to be paid in death case or at insurance contract maturity if it is a endowments insurance, interest rate derived from the investments of premiums, the expenses related to the issuance of the insurance contract, the insurance company's profit.

This technique of determining the insurance premiums and mathematical reserves in life insurance is particularly important. Correct determination of premiums

* *Ph.D. Candidate, University of Craiova, Romania, ana_preda79@yahoo.com*
Ph.D. Candidate, University of Craiova, Romania

is based on observance of some clear principles (Ciurel, 2000): insurance premiums must be adequate, which means that for a group of contracts, the money collected from policyholders, plus the interest earned from the investment of these amounts, shall be sufficient to pay all promised amounts and cover the insurance company expenses; insurance premiums must be equitable, that risk must consider each person insured; insurance premiums should not be excessive compared to the insured sums. I will analyze the price of three types of insurance policy**:** survival insurance, whole life insurance, mixed life insurance.

Survival insurance is characterized by the fact that the insurer undertakes to pay the insured sum when the contract expires provided that the insured is alive. Sum insured consists from insurance premiums paid by the insured during the insurance period, accumulated and capitalized in various ways by the insurer. So, the insured receive the insured sum only if, on expiry of the contract, is alive, otherwise "the insurer is considered released from that service which is stated in the insurance contract, having no obligation to the heirs of the insured" (Dănulețiu, 2007) Survival insurance is not attractive, so does not stimulate saving.

Whole life insurance aims to protect the insured against the risk of death, a future risk, sure, but uncertain as time. A whole life insurance has a limited duration and requires that insurer pays to the beneficiary the sum insured if the insured's death occurred during the period of the contract. Instead if the insured survives when the contract expires, then the insurer is relieved of any obligation to the insured, "insurance premiums are set according to the characteristics of each contract based on actuarial calculations" (Alexandru & Armeanu, 2003), whole life insurance is an insurance protection against the risk determined and not a savings insurance.

Mixed life insurance is characterized by the fact that "the insurer offers a product that covers both risks through a single contract, survival and death." (Badea & Ionescu, 2001). Coverage in an insurance contract the two alternative risks not remove their contradictory character. It seems that the insured is protected in both cases. In fact to the death of the insured, insurance beneficiary shall receive the sum insured and if the insured survives, he benefits of the sum insured. If the insured is protected for both risks, he pays the insurance premium related to both risk of death and survival, cumulatively, the insurer actually justified by the requirement to achieve a balance between premiums earned and benefits incurred.

2. CASE STUDIES CONCERNING THE NET UNIQUE PREMIUM

I studied the unique net premium for a **survival insurance**. These types of insurance have the following technical characteristics: insurance period denoted by n, expressed in years; insured sum, denoted by S;

Risks covered by such insurance may be: if the insured to age x, survives to age x + n years (= end insurance), the insurance company pays to this person the sum S at the end of the insurance period; if the insured to age x does not survives to age x + n years, then the insurance company does not pay the sum S, and no premium refund;

The insured sum can be paid by the insurer: lump sum settlement or payment in driblets. The unique net premium for a survival insurance, the insured sum paid with lump sum settlement, is calculated as:

$$ {}_nE_x \equiv A^l_{xn} = \frac{l_{x+n} \cdot v^n}{l_x} \qquad (1) $$

Where: ${}_nE_x \equiv A^1_{xn}$ - the unique net premium with unitary sum insured; l_x- number of survivors to age x; n- period of insurance, years; v^n- life actualization factor.

We both denominator and numerator weighting of v^n and we get:

$$ {}_nE_x \equiv A^l_{xn} = \frac{l_{x+n} \cdot v^n}{l_x} \cdot \frac{v^x}{v^x} = \frac{l_{x+n} \cdot v^{x+n}}{l_x \cdot v^x} \qquad (2) $$

The relation l_x . v^x =D_x,(D_x - are commutation numbers) so then:

$$ A^l_{xn} = \frac{D_{x+n}}{D_x} \qquad (3) $$

When the insured sum S is other than unitary, the premium is calculated:

$$ P = S \cdot A^l_{xn} = S \cdot \frac{D_{x+n}}{D_x} \qquad (4) $$

For example, we calculate the net premium to be paid by a 30 years old person, who buy a survival insurance policy, period contract 20 years, with a sum insured of 10000 c.u., the interest rate is 10%. From mortality tables we have: D_{50}=716; D_{30}=5403; P=10000$\frac{716}{5403}$=1325,18 c.u.

Besides survival insurance, the most popular and the cheapest are considered the **whole life insurance**. The insurer will pay to the insurance beneficiary a certain amount of money at the insured person's death. Depending on contract period these insurance can be: - annuity insurance where the premium is denoted by A_x; term insurance where the premium is denoted by $A^1_{x:n}$.

The first type of insurance supposes the insurer obligation to pay to the insurance beneficiary a sum of money upon the death of the insured person after any period when death happens. The payment of sum insured can be at the end of year when the insured died or immediately after death. In calculating the unique net premium for a whole life insurance annuity, with payment of sum insured at the end of year of insured death are taken into account: the insurer suppose that all persons lx of age x buy a whole life insurance an indefinite period. From the mortality table we know that during the first year of contract, will die d*x* people and will remain alive l*x* +1, so the insurer will pay d*x* c.u.

Since payment will be made at the end of death year, the present value of payments in the first year will be $d_x \cdot v$, in the second year $d_{x+1} \cdot v^2$, in the third year $d_{x+1} \cdot v^3$, etc., in this case the present value of total payments in - x + 1 years are:

$$d_x \cdot v + d_{x+1} \cdot v^2 + ... + d_\omega \cdot v^{\omega - x + 1} \qquad (5)$$

$$A_x = \frac{d_x \cdot v + d_{x+1} \cdot v^2 + ... + d_\omega \cdot v^{\omega - x + 1}}{l_x} \qquad (6)$$

To simplify the calculations we multiply both denominator and numerator with v^x and $l_x . v^x$ is denoted by D_x and $d_x \cdot v^{x+1} = C_x$:

$$A_x = \frac{C_x + C_{x+1} + ... + C_\omega}{D_x} = \frac{\sum_{t=x}^{\omega} C_t}{D_x} \qquad (7)$$

The numerator is calculated and represent M_x (M_x - are commutation numbers):

$$A_x = \frac{M_x}{D_x} \qquad (8) \qquad\qquad P = S \cdot A_x = S \cdot \frac{M_x}{D_x} \qquad (9)$$

When the insured sum S is other than unitary, the premium is calculated according to rel. (9). If the payment of sum insured is made immediately after insured death the insurance premium is denoted by $\bar{A}_x$ (D_x, M_x - are commutation numbers, i - interest rate):

$$\bar{A}_x = \frac{M_x}{D_x} \cdot \sqrt{1+i} \qquad (10)$$

I will exemplify these types of premiums using the following data: a 30 years old person, who buys a annuity insurance policy, sum insured 10000 c.u., annual interest rate is 10%.

- the sum insured will be received at the year end of death:

$$A_{30} = \frac{M_{30}}{D_{30}} = \frac{328}{5403} = 0{,}0607 \text{ , } P = 607 \text{ c.u.;}$$

- the sum insured will be paid immediately after death:

- $\bar{A} = \frac{M_{30}}{D_{30}} \cdot \sqrt{1+0{,}1} = 0{,}060 \cdot 1{,}048 = 0{,}0628$, P = 628 lei

The second type of contract, term insurance, assumes the obligation to pay a sum of money to insurance beneficiary after the insured death, if his death occurs within the period specified in the insurance contract. The reasoning behind the premium calculation is the same as the annuity insurance.

$$A^{1}_{x:n} = \frac{M_x - M_{x+n}}{D_x} \tag{11}$$

When the insured sum is other than unitary:

$$\mathrm{P} = \mathrm{S} \cdot A^{1}_{x:n} = S \cdot \frac{M_x - M_{x+n}}{D_x} \tag{12}$$

Using the dates above for example, if the insured would have bought an insurance policy, contract period - 5 years:

$$\mathrm{A}^{1}_{30:5} = \frac{\mathrm{M}_{30} - \mathrm{M}_{35}}{\mathrm{D}_{30}} = \frac{52}{5403} = 0{,}00962\text{ , P = 96,2 lei.}$$

Next, I determine the unique net premium for a **mixed life insurance**, denoted by $A_{x:n}$ and is calculated by adding net unique premiums of survival insurance and whole life insurance:

$$A_{x:n} = \frac{D_{x+n}}{D_x} + \frac{M_x - M_{x+n}}{D_x} \tag{13}$$

When the insured sum is other than unitary:

$$\mathrm{P} = S \cdot \frac{D_{x+n} + M_x - M_{x+n}}{D_x} \tag{14}$$

I will exemplify using the following data: a 30 years old person, contract period - 5 years, sum insured 10000 c.u., annual interest rate is of 10%.

$$\mathrm{A}_{30:5} = \frac{\mathrm{D}_{35} + \mathrm{M}_{30} - \mathrm{M}_{35}}{\mathrm{D}_{30}} = 0{,}6224,\quad \mathrm{P}=6224;$$

3. CASE STUDIES CONCERNING THE NET ANNUAL PREMIUM

The contract period for a life insurance is for several years and using the net unique premium is extremely rare, because it requires too much financial effort for insured, therefore can be used instalments throughout all contract period, or a shorter period. We talk about net annual insurance premium of survival insurance, a net annual insurance premium of whole life insurance and annual net premium of mixed life insurance.

In case of **survival insurance** we have two possibilities:

- if the payment period of net annual premium coincide with the contract period (m = n), the annual net premium for a person age x, which requires a survival insurance for a period of n years is:

$$P^{1}_{x:n} = \frac{A^{1}_{x:n}}{\ddot{a}_{x:n}} \quad (15) \qquad P^{1}_{x:n} = \frac{D_{x+n}}{N_x - N_{x+n}} \quad (16)$$

Where: $P^{1}_{x:n}$ -the net annual premium; $A^{1}_{x:n}$ -the net unique premium for a survival insurance, $\ddot{a}_{x:n}$ - the net unique premium related to anticipated annuity insurance for a specified period n, D_x, N_x - are commutation numbers.

- if the payment period of net annual premium is less than the contract period (m < n), the annual net premium for a person age x, which requires a survival insurance for a period of n years is:

$$P^{1}_{x:n} = \frac{A^{1}_{x:n}}{\ddot{a}_{x:m}} \quad (17) \qquad P^{1}_{x:n} = \frac{D_{x+n}}{N_x - N_{x:m}} \quad (18)$$

I calculated the net annual premium that should be paid by a 30 years old person, which requires a survival insurance, contract period 5 years, sum insured 10000 c.u., annual interest rate 10%:

1. m = n, $\mathrm{P}^{1}_{30:5} = \frac{\mathrm{D}_{35}}{\mathrm{N}_{30} - \mathrm{N}_{35}} = \frac{3311}{55822 - 33385} = 0{,}1475$ P=1475 c.u.;

2. m < n, if the insured wants to pay the premiums in 3 years:

$\mathrm{P}^{1}_{\mathrm{x:n}} = \frac{\mathrm{D}_{35}}{\mathrm{N}_{30} - \mathrm{N}_{33}} = \frac{3311}{55822 - 41070} = 0{,}2244$ P=2244 c.u.;

Relating to **whole life insurance** I presented the net annual premium for:
- annuity insurance where the premium is denoted by P_x;
- term insurance where the premium is denoted by P_{xn};

For the first type of insurance we have two situations:

1. if the payment period of net annual premium coincide with the contract period, m = n:

$$P_x = \frac{A_x}{\ddot{a}_x} \quad (19) \qquad P_x = \frac{M_x}{N_x} \quad (20)$$

Where: A_x - the net unique premium for a annuity insurance, $\ddot{a}_x$ - the net unique premium related to an anticipated annuity insurance,

M_x, N_x - are commutation numbers

2. if the payment period of net annual premium is less than the contract period m < n:

$$P_x = \frac{A_x}{\ddot{a}_{xm}} \quad (21) \qquad P_x = \frac{M_x}{N_x - N_{x+m}} \quad (22)$$

For the second type of insurance we have, also, two situations:

1. m = n

$$P^1_{x:n} = \frac{A^1_{x:n}}{\ddot{a}_{x:n}} \quad (23) \qquad P^1_{x:n} = \frac{M_x - M_{x+n}}{N_x - N_{x+n}} \quad (24)$$

Where: $A^1_{x:n}$ - the net unique premium for a term insurance (whole life insurance), $\ddot{a}_{x:n}$ -the net unique premium related to an anticipated term insurance; M_x, N_x - are commutation numbers

2. m< n

$$P^1_{x:n} = \frac{A^1_{x:n}}{\ddot{a}_{x:m}} \quad (25) \qquad P^1_{xn} = \frac{M_x - M_{x+n}}{N_x - N_{x+m}} \quad (26)$$

I calculated the net annual premium that should be paid by a 30 years old person, which requires a whole life insurance, sum insured 10000 c.u., annual interest rate 10%:

1. annuity insurance
   - m=n

$$P_{30} = \frac{M_{30}}{N_{30}} = \frac{328}{55822} = 0{,}0058 \text{ , } \quad \text{P=58 c.u.;}$$

- m < n, the insured wishes to pay the premium in 10 years:

$$\mathrm{P}_{30} = \frac{\mathrm{M}_{30}}{\mathrm{N}_{30} - \mathrm{N}_{40}} = \frac{328}{55822 - 19670} = 0{,}0090 \quad \text{P=90 c.u.;}$$

2. term insurance
   - m= n= 5

$$\mathrm{P}^1_{30:5} = \frac{\mathrm{M}_{30} - \mathrm{M}_{35}}{\mathrm{N}_{30} - \mathrm{N}_{35}} = \frac{328 - 276}{55822 - 33385} = 0{,}0023 \quad \text{P=23 c.u.;}$$

- m < n the insured wishes to pay the premium in 3 years:

$$\mathrm{P}^1_{30:5} = \frac{\mathrm{M}_{30} - \mathrm{M}_{35}}{\mathrm{N}_{30} - \mathrm{N}_{33}} = \frac{328 - 276}{55822 - 41070} = 0{,}0035 \quad \text{P=35 c.u.;}$$

The net annual premium for a **mixed life insurance** is calculated by adding net annual premiums of survival insurance and whole life insurance:

$$P_{x:n} = P^1_{x:n} + P^1_{x:n} = \frac{A^1_{x:n}}{\ddot{a}_{x:n}} + \frac{A^1_{x:n}}{\ddot{a}_{x:n}} \quad (27) \qquad P_{x:n} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} \quad (28)$$

when m < n the formula became:

$$P_{x:n} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+m}} \qquad (29)$$

I calculated the net annual premium that should be paid by a 30 years old person, which requires a mixed life insurance, sum insured 10000 c.u., annual interest rate is 10%:

- m = n= 5, $P_{30:5} = P^1_{30:5} + P^1_{30:5}$ =0,1475+0,0023=0,1498, P=1498 c.u.;
- m < n = 3

$$P_{30:5} = \frac{M_{30} - M_{35} + D_{35}}{N_{30} - N_{33}} = \frac{328 - 276 + 3311}{55822 - 41070} = 0{,}2279 \quad \text{P=2279 c.u.}$$

4. CONCLUSIONS

The above net premiums presented guarantees only cover obligations to the insured, insurance payments, when happen the insured risk, but in order to cover costs of conducting insurance operations, the insurers add an addition to net premium thus obtaining gross premium, paid by the insured when purchasing the insurance policy. Insurer created on account of premiums received from policyholders, a fund which is capitalized as bank deposits or other investments in economic cycle and produce incomes that satisfy insurer's obligations to policyholders, the insured sum payment. Insurance premiums must be adequate, equitable, should not be excessive compared to the insured sums, so that show haw important is the actuary activity. The researcher's analysis show that in all developed countries in the world, males have higher overall rates of mortality than females, so for premium calculation must be taken into discussion other important factors: including whether or not a person smokes, their age and marital status, where they live and their lifestyle in general. The same thing we can say about female life expectancy exceeds that of males today, but this has been the case in these countries for many years.

REFERENCES:

[1]. **Alexandru, F.; Armeanu, D.** (2003) *Asigurări de bunuri şi persoane*, Editura Economică, Bucureşti

[2]. **Armeanu, D.** (2008) *Metode şi tehnici de asigurare*, Editura Universității Româno-Britanice, Bucureşti

[3]. **Anghelache, C.V.** (2006) *Analiză actuarială în asigurări*, Editura Economică, Bucureşti

[4]. **Badea, D.; Ionescu, L.** (2001) *Asigurările de persoane și reflectarea lor în contabilitate*, Editura Economică, București

[5]. **Ciurel, V.** (2000) *Asigurări și reasigurări:abordări teoretice și practici internaționale*, Editura All-Beck, București

[6]. **Dănulețiu, D.C.** (2007) *Asigurări comerciale*, Editura Risoprint, Cluj Napoca