INCOME DISTRIBUTION AND RENT SEEKING COSTS: A NOTE

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ABSTRACT: *This paper analyzes a rent seeking competition where risk neutral agents have a different income. The individual income is private information but the income distribution is common information. Like the Hillman and Samet (1987) model the individual outlays in rent seeking is equal to the expected rent, but the winning probability for each agent is a function of its income. As a consequence, the social costs of the rent seeking depend on the income distribution and they can be lesser than the traditional measure pointed out by Hillman and Samet (1987).*

KEY WORDS: *rent-seeking; dissipation of rent, income distribution*

JEL CLASSIFICATION: D72; D31; D78

1. INTRODUCTION

The literature on the rent seeking (Posner 1975; Tullock 1980; Hillman and Katz, 1984) is very extensive. Particular attention is posed on the social costs of the rent seeking activities that is on the amount of resources spent for unproductive lobbying activities and thus subtracted from productive activities. In technical language, when the total amount of invested resources is equal to the value of the rent, a total dissipation occurs with no added value for the economic system.

As the literature points out, the phenomenon verifies in different circumstances: regulation process (Stigler, 1971), patent competition in the R&D, job promotion within organizations (Milgrom and Roberts, 1993), application of quotas and of favourable legislation in agricultural sector (Bachev, 2009), auctions (Baye et al., 1996; Amann and Leininger, 1996; Che and Gale, 1998).

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Following Hillman and Samet (1987), we analyze a rent seeking competition between two risk neutral agents. The rent seekers agents are asymmetric because they have different income and the rent seeking competition is structured as a first price auction. In this framework the paper shows that the social costs of the rent seeking crucially depend on the income distribution.

The paper is structured as follows. Section 2 is a review of the literature, sections 3 and 3.1 describe the rent seeking competition and the expenditure functions of the agents. The maximization problem of the agents is analyzed in section 4 while the social costs of the rent seeking are calculated in section 5. Finally, Section 6 concludes the paper.

2. LITERATURE OVERVIEW

Public choice literature on rent seeking is very extensive. Different models are based on different hypotheses. Some models consider risk neutral rent seekers agents competing for an exogenous rent X (Posner 1975; Tullock 1980 a); 1980 b); 1980 c); 1989), while others models analyze the rent seeking competition with risk averse rent seekers agents (Hillman and Katz 1984).

Unlike the Posner's model where there is a total dissipation of the rent, Tullock highlights a different result. In his model the probability of each agent to obtain the rent is not a linear function of investments in rent seeking activities, but it depends on a parameter (r) representing the "effectiveness" of the rent seeking expenditures.

Therefore, in this setting the value of r (equal for all agents) and the number of the agents are crucial to determine the amount of resources invested in lobbying activities and representing the "social costs of the rent seeking". When the total amount of individual expenditures in rent seeking is equal to the value of the rent for which the agents compete, a total dissipation of the rent occurs. Extending the Tullock's model, Dari-Mattiacci and Parisi (2005) show that, introducing an exit option for risk neutral rent seeker agents, valuable rents may remain unexploited[1]. These lost rents represent an additional social cost of the rent seeking.

The case of risk adverse agents is considered by Hillman and Katz (1984). In their paper the social cost of the rent seeking is lower than the risk neutrality case if the coefficient of absolute risk aversion is decreasing.

Another class of models (Krueger 1974; Appelbaum and Katz 1987; Fabella 1996) considers the rent endogenously determined by the rent seeking competition and analyzes the behavior of the rent seekers to obtain this rent. On the contrary, Appelbaum and Katz (1987) present an analysis of the interaction of a regulator and the rent seeker firms and they show that the outcome (the value of the rent and the rent seeking expenditures) depends on the relative bargaining powers of the regulator and the firms.

In any case, all these models are based on the crucial assumption that the agents are identical: they have the same utility function, the same income and solve the same maximization problem. Therefore, the equilibria of the rent seeking competition

[1] See also J. Munster (2007).

are necessarily symmetrical and the social cost of the rent seeking is the product between the individual expenditure in lobbying activities and the number of the agents. A different perspective appears if asymmetrical agents are considered (Rogerson 1982; Leininger 1993; Kohli and Singh 1999).

In particular, Kohli and Singh (1999) generalize the models of Tullock (1980) and of Appelbaum and Katz (1987). Unlike Appelbaum and Katz (1987), they introduce an asymmetry among the agents rent seekers captured by a parameter representing the different effectiveness of the lobbying outlays influencing the winning probability (i.e. the probability to obtain the rent). Therefore, even if the agents invest the same amount of resources, they can produce a different level of "influence" towards the decision maker and so, a different probability to obtain the rent. In this sense, they generalize the Tullock's model (1980) considering agents characterized by a different parameter of effectiveness "r".

Other extensions (Leininger (1992) consider an interaction à la Stackelberg between rent seeker firms. In this case, the outcome of the rent seeking game depends on the asymmetries in effectiveness and on timing of the moves.

Finally, recently the auction theory focuses on game theory methodology to model rent seeking competition. In particular, the "all pay" first price models are linked to the rent seeking framework because, unlike a standard auction where active participation constitutes only a conditional commitment, in a "all pay" first price auction active participation constitutes an unconditional commitment (even the losers pay).

In this class of models (Baye et al., 1996; Amann and Leininger, 1996) the asymmetry among competitive agents is related to the subjective valuation of the bid's object.

3. THE RENT SEEKING COMPETITION

We consider two agents risk neutral. At the same time they compete for a rent X[2] investing some resources in rent seeking outlays. The assignment mechanism is like a first price auction: the agent with a greater bid obtains the rent, but all competing agents bear sunk costs.

The agents are not identical because they have a different income, respectively W_1 and W_2. The individual level of income identifies the *type* of the agent and it is a continuous random variable on $[0;\overline{W}]$. Therefore, $[0;\overline{W}]$ is the set of the possible *types*. Each agent only knows his /her type (his/her income). So, the agent's type is a private information, but the density function, p(w), of the variable W and its cumulative distribution, P(W), are common information. The standard assumptions on cumulative distribution are satisfied so that: P(0)=0 and $P(\overline{W})=1$. W_1 and W_2 are independent events.

[2] The analysis is restricted to the short run, since other agents can not enter to compete for this rent.

In this framework, we consider the rent seeking outlays as a function of the income[3]. The following section clarifies this point.

3.1. The expenditure functions

Each agent's expenditure in rent seeking is a continuous increasing function of the income: $s_i = f(W_i)$ with $f' > 0$ and i= 1,2

The asymmetry between the agents is represented only by the different individual income. They solve the same maximization problem, have the same risk attitude, the same functional form of the function f(.)[4].
Since W_i is a random variable, a distribution of probability is induced on $s_i \in [0; \bar{s} = f(\bar{W})]$. In order to obtain the density function of s_i, a transformation of random variable is needed.

Let G(s) be the probability that the expenditure $s_i = f(W_i)$ is lower than a given s:

$$G(s)=\text{prob}\{f(W_i) < s\}=\text{prob}\{W_i < f^{-1}(s)\} =$$
$$= \int_0^{f^{-1}(s)} p(W_i)dW_i = P[f^{-1}(s)] - P(0) = P[f^{-1}(s)] \qquad (1)$$

The density function g(s) is[5]: $g(s)= p(f^{-1}(s))\cdot \frac{df^{-1}(s)}{ds} = p(f^{-1}(s))\cdot (f^{-1}(s))'$

Given the assignment mechanism of the rent, the agent 1 obtains the rent if his/her outlay in rent seeking ($s_1 = f(W_1)$) is greater than the opponent's outlay ($s_2 = f(W_2)$). In other words, the winning probability of the agent 1 is the probability that his/her income is greater than the opponent's income [6]:

$$\text{prob}\{s_2 < s_1\}= \text{prob}\{f(W_2) < s_1\}= \text{prob}\{W_2 < f^{-1}(s_1)\} =$$
$$= \int_0^{f^{-1}(s_1)} p(W_2)dW_2 = P[f^{-1}(s_1)]^{7} \qquad (2)$$

The losing probability for the agent 1 is $\{1 - P[f^{-1}(s_1)]\}$.

4. THE MAXIMIZATION PROBLEM OF THE AGENTS

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[3]In most of the models of rent seeking,the rent seeking expenditures depend on the value of the rent (Tullock '67; Krueger '74; Posner '75; Tullock '80).

[4] In the contrary case, it is obvious that a more risk adverse agent invests less than the other agent.

[5] It is the first derivative with respect to s of the cumulative distribution G(s). Since G(s) is a composite function of s, the first derivative is obtained applying the chain rule.

[6]Because the expenditure functions are increasing in income.

[7] Since $f^{-1}(s_1) = W_1$, $P[f^{-1}(s_1)] = P(W_1)$.Nevertheless the transformation of random variable is needed because even if the new random variable s(.) has the same probability order of w (because it is a monotone increasing transformation of W), the level of probability associated to s(.) and W can be different.

Each agent maximizes his/her payoff, i.e. his/her net income given by the personal income (Wi with i=1,2) minus the expenditures in rent seeking (s_i with i=1,2) plus the rent (X), if obtained.

Given the risk neutrality hypothesis, the individual participation constraint to the rent seeking competition is such that the expected payoff is greater or equal to the initial income:

$$(W_i + X - s_i)\, P[\, f^{-1}(s_i)] + (W_i - s_i)\, \{1 - P[\, f^{-1}(s_i)]\} \geq W_i \quad \text{for i=1, 2} \qquad (3)$$

With simple calculations, the (3) can be rewritten as:

$$s_i \leq XP\left[f^{-1}(s_i)\right] \quad \text{for i= 1, 2} \qquad (3')$$

The condition (3') represents the set of the possible strategies for each agent. The individual outlays in rent seeking are smaller or equal to the expected rent[8]. The agent 1 solves the following maximization problem (the identical problem is solved by the agent 2 changing the subscripts):

$$\max_{s_1} E(U_1) = (W_1 + X - s_1)\, P[\, f^{-1}(s_1)] + (W_1 - s_1)\, \{1 - P[\, f^{-1}(s_1)]\} \qquad (4)$$

With simple manipulations, we can write: $\max_{s_1} E(U_1) = X\, P[\, f^{-1}(s_1)] + W_1 - s_1$

The first order condition is: $\frac{dE(U_1)}{ds_1} = X\, p(f^{-1}(s_1))\cdot(f^{-1}(s_1))' - 1 = 0$. Since $s_1 = f(W_1)$ and $W_1 = f^{-1}(s_1)$, we have that $f' = \frac{ds_1}{dW_1}$ and $(f^{-1})' = \frac{dW_1}{ds_1} = \frac{1}{f'}$, the first order condition becomes:

$$\frac{dE(U_1)}{ds_1} = -1 + X\, p(W_1)\cdot \frac{1}{f'} = 0 \qquad (5)$$

or

$$Xp(W_1) = f' \qquad (5')$$

Integrating previous expression, we have the expenditure function of the agent 1:

$$f(W_1) = XP(W_1) \qquad (6)$$

where
$P(W_1)$ is the winning probability of the agent 1 and represents the probability that the opponent competitor has a smaller income.

The expenditure function for the agent 2 is:

$$f(W_2) = XP(W_2) \qquad (6')$$

[8] Following the traditional framework of the public choice models, we assume that the agent is not budget constrained or, in other words, that the agent can borrow.

where
$P(W_2)$ is the winning probability of the agent 2.

Like all the rent seeking models with risk neutral agents, each agent invests in rent seeking activities an amount of resources equal to the expected value of the rent.

This is not surprising and crucially derives from the risk neutrality of the agents. Nevertheless, in most of the models considering symmetrical agents, the winning probability is $\frac{1}{n}$ with n= 1,2,....n and each agent competes for an expected rent equal to $\frac{1}{n}$ X. In the considered case, the expected value of the rent is not the same for the two agents, because they have a different winning probability depending from their income. So for the agent 1, given his/her income of W_1, the winning probability is the probability that the unknown variable W_2 is less than W_1. The symmetrical reasoning applies to the agent 2. Obviously, the agent with a greater income invests a greater amount of resources in rent seeking activities.

5. THE SOCIAL COSTS OF THE RENT SEEKING

Following the literature, we define the social costs (SC) of the rent seeking activities the total amount of resources invested in unproductive lobbying activities and so, subtracted from productive activities.
In this case:

$$SC= s_1 + s_2 = f(W_1) + f(W_2) = XP(W_1) + XP(W_2)$$

There is a total dissipation of the rent if the total amount of resources invested are equal to the value of the rent, that is:

$XP(W_1) + XP(W_2) = X$

The previous expression can written as

$$P(W_1) + P(W_2) = 1 \quad (7)$$ [9]

or

$$P(W_1) + P(W_2) = P(\overline{W})$$

With identical agents ($W_1 = W_2 = W$), the "(6)" is 2P(W)=1 or P(W) = $\frac{1}{2}$. Therefore, a total dissipation of the rent results when both agents are *median* agents,

[9] The "(6)" can be written as $P(W_1) = 1-P(W_2)$. Note that $P(W_1)$ is the probability that $W_2 < W_1$ for the agent 1 and $P(W_2)$ is the probability that $W_1 < W_2$ for the agent 2. Then, 1- $P(W_2)$ is the probability that $W_2 < W_1$. Nevertheless, $P(W_1)$ is not necessarily equal to 1- $P(W_2)$ because W_1 and W_2 are private information and so, $P(W_1)$ and $P(W_2)$ are probability valuated by the agents 1 and 2 given one's own income. Then, a total dissipation of the rent results only if the valuated probability $P(W_1)$ and 1- $P(W_2)$ are equal. On the contrary, it is possible an underdissipation or an overdissipation as summarized by the expressions "(7)" and "(8)".

i.e. they have the median income. With asymmetrical agents ($W_1 \neq W_2$) a total dissipation of the rent can result even if the variation between the two incomes is high. This point is summarized in the following claim:

Claim 1

If the agents rent seekers are characterized by a different income, the rent seeking activity implies a total dissipation of the rent if the sum of the cumulative probability associated to their incomes is equal to one.

At the same way we have an underdissipation and an overdissipation of the rent if:

$$P(W_1) + P(W_2) < 1 \quad (8)$$

$$P(W_1) + P(W_2) > 1 \quad (9)$$

Intuitively, claim 1 says that when both the agents have an income smaller or greater than the median income, conditions (8) and (9) are respectively satisfied confirming the underdissipation and the overdissipation of the rent. When the agents rent seekers are in opposite position with respect to the median income, the social costs of the rent seeking crucially depend on the income distribution of the rent seekers agents[10].

6. CONCLUSIONS

This paper analyzes a rent seeking competition with risk neutral agents having a different income whose distribution is known. The expenditure functions are increasing with respect to the income. In this framework, the analysis shows that the social costs of the rent seeking depend on the income distribution. In particular, unlike the Hillman and Samet's result (1987), the rent seeking costs are smaller than the rent if the sum of the cumulative probabilities associated to the individual incomes is lesser than one.

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