CREDIT RISK. DETERMINATION MODELS

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ABSTRACT: *The internationalization of financial flows and banking and the rapid development of markets have changed the financial sector, causing him to respond with force and imagination. Under these conditions, the concerns of financial and banking institutions, rating institutions are increasingly turning to find the best solutions to hedge risks and maximize profits. This paper aims to present a number of advantages, but also limits the Merton model, the first structural model for modeling credit risk. Also, some are extensions of the model, some empirical research and performance known, others such as state-dependent models (SDM), which together with the liquidation process models (LPM), are two recent efforts in the structural models, show different phenomena in real life.*

KEY WORDS: *credit risk*; *structural model; payment incapacity; financiar tools; the theory of evaluation for option price.*

JEL CLASSIFICATION: *C10, G1.*

1. INTRODUCTION

The Portfolio Manager model was first developed by Robert C. Merton [1] shortly after the theory that evaluates the price of the options of Black and Scholes[2].

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[1] Merton Robert Cox – born 31 July 1944, is the American economist, Nobel Prize winner, along with Myron Scoles for economics in 1997 for their papers in the field of options. For the first time the model was designed and appeared in literature starting with the year 1974, through out the article published by Robert Merton and subsequently developed by Galai and Masulis (1976)

[2] Black Fischer and Schole Myron - in 1973, the two researchers developed a mathematical model of the capital market, in which the price of the capital is a stochastic process called "The Model which evaluates the price of the option". In 1997 Myron Schole and Robert C. Merton received the Nobel prize for economics for their contribution in the field of financial instruments market

This is a structural model, which uses the evolution of structural variables of a company, such as the value of assets and liabilities, to determine the duration of time required to reach the moment when bankruptcy is triggered. The Merton model first appeared in a research paper in 1974 and is considered the first modern model of calculating the probability of entering into bankruptcy, and also the first structural model.

The Merton model tells us that if a company is unable to pay its debts, at maturity, the value of the company's assets will be lower than the value of its debts. Suppose that the company's capital structure is composed of a capital ***E*** and a long-term financing in the form of a bond issue. For example, we take, a bond with zero-coupon, with a maturity ***T***, and the nominal value ***D***. Merton Believed that if at the moment **T**, $V_T < D$, then the company will be in bankruptcy, at least theoretically, and the market value of the capital will be 0. . In this case the creditors will gain control of the company and the shareholders will receive nothing.

If, at the same time, $V_T > D$,then the company will pay its debts, then the loan will be repaid according to the schedule for repayment, and the value of the company's capital at the moment ***T*** is: $V_T - D$.

Becomes logical the next relationship:

$$E_T = \max(V_T - D, 0) \qquad (1)$$

The model assumes that the amount of capital of a company, E_T, can be considered a "call" option with the value of the market of the company as an active support, V_T, and the exercise price shall be assimilated to the nominal value of the debt, *D*.

In these circumstances, the capital of the company at a time **T** is a function which depends on several variables, such as the value of the market at that time, the exercise price, the volatility and the interest rate.

Then, $E_t = f(V_t, D, \sigma, r, T - t)$, where V_t and σ_V are unobservable variables.

When we apply the formula that evaluates the price of the options Black-Scholes (1973), results:

$$E_0 = V_0 \times N(d_1) - D \times e^{-rT} \times N(d_2), \qquad (2)$$

where:

$$d_1 = \frac{\ln \frac{V_0}{D} + (r + \frac{\sigma_V^{\,2}}{2}) \times T}{\sigma_V \sqrt{T}} \qquad (3)$$

$$d_2 = d_1 - \sigma_V \times T \qquad (4)$$

The market value of the loan is $V_0 - E_0$.

The neutral risk probability that the company to go bankrupt is $N(-d_2)$, but for its calculation is needed V_0 and σ_V , which are unobservable variables. But if the company is traded on the exchange market, the value of the capital can be seen, E_0 and the volatility of the capital can be estimated σ_E :

$$\sigma_E \times E_0 = \frac{\partial E}{\partial V} \times \sigma_V \times V_0 \qquad (5)$$

or

$$\sigma_E \times E_0 = N(d_1) \times \sigma_V \times V_0 \qquad (6)$$

If we build a system of two simultaneous equations based on this relationship and the equation of options price, it can be solved for obtaining the values V_0 and σ_V . This model was developed some years later by other two researchers, Gala and Masulis (1976), who believe that the risk of credit can be covered, if the "call" option is replaced by purchasing a "put" option, P, which has the value of the firm as an active support, V, as the exercise price, the nominal value of companies debt, D, and maturity T. By purchasing this option, the creditor of the company eliminates completely the loan risk and makes the initial loan, which was evaluated as a loan with high risk, into a risk free loan with the nominal value D. To calculate the market value of the loan we still apply the formula that evaluates the price of the options Black-Scholes (1973).

2. ADVANTAGES AND LIMITATIONS OF THE MODEL

Abel Elizalde[3] presents in his paper Credit Risk Models II the main advantages and limits of the Merton model, taking into account the contributions and the subsequent developments in this field which, as the products change and develop, show other aspects of the risk of entering into bankruptcy.

The main advantage of the Merton model is that it allows the application of theory that evaluates the price of the options Black and Scholes. But in order to do this, the model must fallow certain assumptions, necessary for adapting to the dynamics of the process of evaluation of companies assets, of the interest rates and of the structure of the capital as required by the Black-Scholes model.

The Merton model is a compromise between realistic assumptions and ease of implementation, and it prefers the last one. All the extensions of this model, introduce more realistic assumptions, trying to finalize a model, not too difficult to implement, with feasible solutions in terms of numbers, which expresses the value of debt and the probabilities of bankruptcy.

[3] Elizalde, A. (2005) *Credit Risk Models II – Structural Models*, CEMF – Working Paper No 0606, April 2006

Merton, itself presents some extensions of the model, by taking into account the coupon bonds, the state bonds, the stochastic interest rates, and another perspective of the Modigliani-Miller[4] theory referring to the principle that he promotes[5].

A shortcoming of the Merton model is this limitation, through which the bankruptcy is register only at the maturity of the debt, which eliminates the alternative that the bankruptcy could be installed earlier than the maturity of the debt, regardless of the value of the companies assets. If the value of the company reaches minimum levels prior to the maturity of the debt, but the company is able to unbend and to pay the payment obligations at maturity, this assumption would mean, according to the Merton model, that bankruptcy is excluded.

Another limitation is the fact that this model sees the capital of a company as a simple bond with a zero coupon, and in reality things are more complex.

Geske[6] develops the model and sees a company's debt structure as a coupon bond, in which each payment of a coupon is treated as a composed option and a possible cause of bankruptcy. At each coupon payment, shareholders have the opportunity either to make coupon payment to bond holders, in which case they obtain the right to control the company until the next payment, or they can choose not to make the payment of the coupon and the company then enters into bankruptcy. Also, Geske extends the model taking into account characteristics such as: the reduction of funds, the safety contracts, the subordinated debt and the payment restrictions.

The assumption of a constant structure of the interest rate and uniform represents another major model which can be taken into consideration. Jones et al. (1984, p. 624) suggests that "there is evidence that the introduction of the stochastic interest rates and also taxes should improve performance of the model." Stochastic interest rates allow the introduction of the correlation between the amount of assets of the company and the interest rate on short-term, which was also reported by Ronn Verma (1986), Kim, Ramaswamy and Sundaresan (1993)[7], Nielsen et al. (1993), Longstaff, Schwartz[8], Eric Briys, Frannois De Varenne[9] (1997) and Hsu, SAA-Requejo and Santa-Clara (2004).

Another feature of the Merton model is the anticipation of bankruptcy. Once the value of the company is modeled as a geometric Brownian motion and bankruptcy can occur only at the maturity of the debt, then, it can be anticipated with great

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[4] Modigliani, F.; Miller, M.H. (1958) *The cost of Capital, Corporation Finance and the Theory of Investment*, American Economic Review, no 48, June 1958

[5] The principal of the irrelevance of the structure of the capital – for which Franco Modigliani and Merton H. Miller were rewarded with the Nobel Prize for economics in 1985. The theorem implies the existence of an efficient market, where there are no taxes, costs of bankruptcies, no information is distorted, so operating on such a market makes the value of a company not to be affected by the way in which it finances itself - capital increase through the issuance of shares or a loan

[6] Geske, R. (1977) *The valuation of corporate Liabilities as compound options*; (1979) *The valuation of compound option*

[7] Kim, I.J.; Ramaswamy, K.; Sundaresan, S. (1993) *Does Default Risk in Coupons Affect the Valuation of Corporate Bonds? A Contingent Claims Model*, Financial Management, Financial Management Association, vol. 22(3)

[8] Longstaff, F.; Schwartz, E. (1995) *Valuing Risky Debt: A New Approach*, Journal of Finance, no 789

[9] Briys, E.; De Varenne, F.; Santomero, M. A. (1997) *On the Risk of Life Insurance Liabilities: Debunking Some Common Pitfalls*

precision that the maturity of the debt is approaching. Therefore, in this approach, bankruptcy does not come as a surprise, so that the models can generate a low credit risk on short term.

Delianedis and Geske (2001) studied how the risk of entering into bankruptcy is influenced by the market value of the instrument of credit, in this case the study is based on a set of corporate bonds.

On this occasion, researchers have concluded that the evolution of the market value of a bond, can influence the risk of bankruptcy to a small extent, the remaining accumulated risk comes from other factors such as taxes, volatility, liquidity risk and the market risk. The volatility is included also in the Merton model, which emphasizes that *"the large variations in the value of the market of the debts, explains a certain size of the growth of bad loans, but this, certainly, isn't the only explanation."*

3. EXTENSIONS OF THE MODEL

In the Merton model, a company reaches payment inability if, at maturity, the value of the companies assets is smaller than the value of the debts.

The second approach in the structural model, was introduced by Black and Cox (1976), in which, the payment inability appears when the value of a company falls below a certain threshold. The paper written by Black and Cox (1976) is the first so-called "First Passage Models" (FPM). The model illustrates the beginning of the bankruptcy, when the value of a companies assets points at a lower barrier, allowing the bankruptcy to appear at any time.

The First Passage Models were introduced by Black and Cox (1976), as an extension of the Merton model, when a company might reach payment inability at any time, not only at the maturity of the debt. Taking into account the fact that the dynamics of the value of a company's assets under the probability of the neutral risk noted with P, are given by the process of diffusion:

$$dV_t = rV_t dt + \sigma_V V_t dW_t \tag{7}$$

and that there is a lower level of the assets so that the company can reach payment inability the once it reaches this level.

Although Black and Cox (1976) considered that the threshold of bankruptcy depends on a period of time, in this model we use a constant, called the threshold of bankruptcy, denoted with K> 0. If we are at the time $t \geq 0$ and the bankruptcy hasn't been triggered yet and Vt> k, then the time at which the company will go bankrupt is noted with τ, and is given by the relationship:

$$\tau = \inf\left(s \geq t / V_s \leq K\right) \tag{8}$$

Using the properties of Brown's motion denoted with Wt, we can conclude that the likelihood of bankruptcy can occur in the period between those two moments, from time t to T is:

$$P\left[\tau \le T / \tau > t\right] = \Phi_{(h)} + \exp\left\{2\left(r - \frac{\sigma_V^2}{2}\right)\ln \left(\frac{K}{V_t}\right)\frac{1}{\sigma_V^2}\right\}\Phi(h_2), \quad (9)$$

where:

$$h_1 = \frac{\ln\left(\frac{K}{e^{r?(?T-t)}V_t}\right) + \frac{\sigma_V^2}{2}(T-t)}{\sigma_V\sqrt{T-t}} \quad (10)$$

$$h_2 = h_1 - \sigma_V\sqrt{T-t} \quad (11)$$

First Passage Models were extended by taking into account some other variables also, such as: stochastic interest rates, bankruptcy costs, taxes, the subordinated debt, the strategic bankruptcy, the stochastic bankruptcy barrier, the volatility of the yield of assets, etc.. Although these developments of the Merton model, bring more realism to the model, they also lead to an increase of its analytical complexity.

The threshold of bankruptcy, always positive, can be interpreted in different ways. We can consider it as a security measure of the company, which allows bond holders to take control of the company, once the value of assets has reached this level.

The safety agreement could act as a protection mechanism for bond holders against poor corporate performance. In this case, the threshold of bankruptcy could be determinant, therefore a non- stochastic process, although it might be time dependent, could have a fixed value and could be determined by exogenous factors when the firm's debts are issued.

If the bankruptcy's barrier is determined by the exogenous factors, as in the studies of Black and Cox (1976) and Longstaff and Schwartz (1995), it acts as a safety barrier to protect shareholders. If the bankruptcy's barrier is determined by the endogenous factors, as a result of shareholders trials to choose the threshold of bankruptcy, it is intended to maximize the company's value[10].

Different structural models have considered the interest rates both as non-stochastic [11] processes and as stochastic [12] processes also, which are time series and can be considered as random functions.

By definition, in the "First Passage Models*",* the inability to pay first appears, when the value of the assets reaches a certain minimal threshold, which is so low that the company is wound up immediately after this state. Also, experts have examined the possibility of the negotiation processes between shareholders and bond holders, if the company goes almost to the point of financial danger, from which the threshold of bankruptcy is a determinant. In the First Passage Models, the threshold of bankruptcy

[10] Leland (1994); Leland & Toft (1996)
[11] Black & Cox (1976); Geske (1977); Leland (1994); Leland & Toft (1996)
[12] Ronn & Verma (1986); Kim, Ramaswamy & Sundaresan (1993); Nielsen et al. (1993); Longstaff & Schwartz (1995); Briys de Varenne (1997); Hsu, ASA-Requejo & Santa Clara (2004)

can be calculated as a weighted arithmetic average of the debts on short term and long term also.

Opposite to this model in recent studies is invoked a new set of theoretical and empirical research, which studies the alternative that appears, if the installation of payment inability doesn't cause the immediate liquidation of the company, but this is beginning of the liquidation process, which could or could not lead to liquidation. This practice maces, for example, with Chapter 11 of U.S.'s bankruptcy law, which, if a company declares bankruptcy procedures, the court shall grant a grace period (which may be several years), in which they can identify the company's financial problems in order to avoid, if possible, the liquidation process. In the study of Abel Elizalde[13] these models are called Liquidation Process Models (LPM).

Another approach of the theoretical structural models is to extend them taking into account that some parameters depend on the state; they are called state-dependent models (SDM). Cash flows, bankruptcy costs and financing costs could be dependent on the state. This branch of structural models can reduce the problems related to the predictability of bankruptcies and can improve the recovery rates, which are the drawbacks of standard models. If we refer to the structural models, the company is subjected to exogenous factors, with the changing of the parameters, that affects the ability to generate cash flows or to cover financing costs, which actually are the main drivers of the likelihood of the bankruptcy.

Hackbarth, Miao and Morellec[14] (2004) and Elizalde, in their paper, published in 2005, presented two models and in both cases, the authors provided expressions used for calculating the equity and debt, whose solutions involves solving ordinary differential equations. In the work presented by Hackbarth, Miao and Morellec the cash flows and the recovery rates depend on the business cycle. The cash flows X_t, follows a geometric Brownian motion and are scaled by a scaling factor of a business cycle. These flows are higher if the economy is expanding $y_H x_t$, than when it is in recession $y_L x_t$, $y_H > y_L$. Similarly, the bankruptcy costs are expressed as a state-dependent portion 1-α of the assets of the company, and in this case, the recovery rate in a growing economy α_H is higher than in a recession α_L, $\alpha_H > \alpha_L$. At every moment in time, there is a probability, determined by exogenous factors, switching between recession and expansion.

The threshold of bankruptcy caused by endogenous factors is elected by the shareholders to maximize the amount of capital and it proves to be higher in recession: bankruptcies of companies are found earlier in recession than in expansion.

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[13] Elizalde, A. (2006) *Credit Risk Models II – Structural Models*, CEMFI Working Paper No.0606, April 2006

[14] Hackbarth, D.; Miao, J.; Morellec, E. (2004) *Capital Structure, Credit Risk, and Macroeconomic Conditions*, FAME Research Paper Series, rp125, International Center for Financial Asset Management and Engineering

CONCLUSION

State dependent models (SDM) are, together with the Liquidation Process Models (LPM), two recent efforts, which, if they are included in the structural models, show different new aspects that, until now, have never been discussed. Although, in theory, these new cases are logical, they have a lack of empirical research and performance testing. SDM assume that some of the parameters that govern the company's ability to generate cash flows or financing costs are dependent on the state, if the countries can represent the business cycle (recession, versus expansion) or the external evaluation of the company.

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